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Nonlinear bilevel programming approach for decentralized supply chain using a hybrid state transition algorithm

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ABSTRACT

This paper investigates a decentralized supply chain that is composed of one manufacturer and multiple distributors. The manufacturer produces goods and wholesales them to multiple distributors and then the distributors sell products to various markets. The entire production period of the manufacturer is divided into several intervals. The decision-making problem of the entire decentralized supply chain is presented as a two-echelon coordination game network, in which each decision-maker can influence decision-making of other levels. A Stackelberg game framework is proposed to coordinate the decision-making process. And then two nonlinear bi-level programming (BLP) models are developed to find the optimal equilibrium decision scheme by switching the leader and follower roles between the manufacturer and the distributors. The models consider the manufacturer's budget constraints in each interval and the market demands are affected by distributors' selling price and advertising strategies. According to the hierarchy and complexity of bi-level programming problem (BLPP), a nested bi-level method based on hybrid state transition algorithm is proposed to address the BLP models, and mapping approximation strategy is utilized to improve computational efficiency. Finally, the numerical experiments are performed to demonstrate the superiority of the proposed method in terms of accuracy and computational efficiency.

1. Introduction

The supply chain consists of independent parties which form a process chain that transforms raw materials into finished products and provides them to the end customer. In the operation of the supply chain, there is competition and cooperation between core enterprises and competitors or upper and lower enterprises at the same time. Increasing competition and globalization of the market motivate enterprises to autonomously participate in the supply chain. Simultaneously, the independence and autonomy of supply chain participants requires that we must look at the supply chain system from a decentralized perspective. The decentralized supply chain is a new type of organizational system whose structure and operating characteristics are different from those of the traditional supply chain, thereby resulting in different cooperative and competitive relationships. Although there are close dependencies between participants in the decentralized supply chain, these enterprises still only care about their own gains and losses instead of considering the overall benefits of the supply chain. Accordingly, each member of the decentralized supply chain has its own goals and the activities of one supply chain member will be hindered by other members. Thence, the decentralized supply chain contains multiple decision-making stages [1–3], which leads decision-making process to present a hierarchical characteristic.

The coordination of decentralized supply chains is the focus of many scholars. Wang et al. [4] designed a cooperation method for decentralized supply chain based on game theory. Combined with the coordinating pricing and inventory replenishment strategies, Boyaci et al. [5] investigated the equilibrium solution when the demand rate is large enough. Davood et al. [6] studied the problem of supplier selection in the decentralized supply chain, and compared the centralized model with the decentralized model. It was shown that decentralized model can obtain a more stable and feasible solution while keeping the total cost unchanged. Haque et al. [7] developed a new bi-level approach to address specific challenges in coordination and sustainability of the decentralized supply chain. They applied dominance and Nash game strategies

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to ensure the coordination of supply and demand throughout the decentralized supply chain network.

Bi-level programming (BLP) is an effective method to address the decentralized management issue with two-level structure [8]. Historically, as a branch of mathematical programming, BLP originated from Stackelberg game theory in the field of economics. In Stackelberg game theory, two decision makers in BLP make decisions one after another, and choose the corresponding strategies according to the other's possible strategies, thereby ensuring that their own benefits are maximized. The final result of the game is to achieve the Nash equilibrium, that is, neither party can get more benefits by adjusting the strategies [9]. The BLP has broadly similar decision-making process. In general, a bi-level programming problem (BLPP) has upper-level leaders and lower-level followers. Members as the leaders make the decision first, and the followers determine their optimal responses based on the decisions of leaders. Then the leaders will adjust the programming according to the decision of the follower, and strive to maximize the profit. This programming process will continuously operate until no decision makers adjust the decision. The entire BLPP contains two interrelated planning problems at the upper and lower levels [10].

Moreover, the BLP method has been widely used to cope with real decentralized decision-making problems. Lu et al. [11, 12] combined linear multi-follower BLP and Kth-best approach. Zheng et al. [13] used the weak linear BLP to model principal–agent problem from a pessimistic perspective. Fan [14], Sinha [15] and Zhang [16] adopted BLP to solve the toll setting problem. Bostian [17] and Xu [18] combined BLP with environmental economics. It is noteworthy that the application of BLP in coordinating the decentralized supply chain has received more and more attention. Yu et al. [19] considered inventory management strategy in one manufacturer–multiple retailers supply chain. They proposed a bi-level model where the manufacturer is the leader and retailers are followers. Amirtaheri et al. [20] investigated a decentralized production–distribution supply chain where the demand is jointly influenced by pricing and advertising policies. According to Stackelberg game theory, they developed two nonlinear BLP models under two power scenarios. Tantiwattanakul et al. [21] studied the decentralized supply chain with multiple products, multiple periods and multiple retailers. They built a nonlinear BLP model to determine the supplier's optimal multi-period wholesale price. Luo et al. [22] modeled the coordination management problem in the decentralized supply chain as a bi-level linear programming problem.

Different BLP models have been proposed for decentralized supply chains, but many of them are linear and cannot accurately reflect most real-life situations. Moreover, although many supply chain coordination mechanisms such as two-echelon coordinated pricing, inventory management, joint advertising, ordering and transportation problems have been investigated by many scholars, most of the existing BLP models focus on designing coordination schemes for single or partial combination problems and cannot achieve effective coordination among decentralized supply chain members. In addition, some key budget constraints have been ignored in the previous models. To conform more to real life and integrate various issues, this paper investigates a decentralized supply chain consisting of one manufacturer and multiple distributors, and establishes two nonlinear BLP models with budget constraints based on Stackelberg game. In the decentralized supply chain, the manufacturer has traditionally been the leader. In recent years, however, the dominant force has shifted from manufacturers to distributors [23]. Consequently, the two proposed BLP models take the manufacturer and the distributors as the leader respectively. In the process of decentralized decision-making, the manufacturer and distributors share relevant decision-making information for the propose of maximizing their profits. Although supply chain members are not directly involved in each other's decision-making process, their decisions can affect the subsequent reactions of other members.

Due to the complex interaction of the upper and lower problems [24], the non-convexity of the search space [25] and the non-differentiability of the objective function, the solution of the BLP models face great challenges. Bard [26] proved that BLPP is an NP-hard problem, and even searching for a local optimal solution of bi-level linear programming is also an NP-hard problem [27]. The traditional methods to solve BLPP can be roughly divided into the following categories: (1) single-level reduction based on Karush–Kuhn–Tucker (KKT) conditions. (2) descent methods. (3) penalty function methods. These methods impose restrictions on the mathematical properties of the BLPP [28], such as convexity and continuous differentiability, so their ability to solve complex nonlinear BLPP (NBLPP) is severely limited. Hence, the intelligent algorithms which do not need to strictly follow the classical assumptions are widely concerned by many researchers. The application of intelligent algorithms to solve BLPP has two forms: (1) single-level reduction scheme. (2) nested scheme. Moreover, due to the use of KKT conditions, single-level reduction scheme still needs to limit the mathematical nature of the lower-level problems. Nested method, relatively, is suitable for a wider range of real-world optimization problems. In order to deal with the BLPP, Huang et al. [29] utilized a method based on nested structure. Wang et al. [30] combined two sole improved fruit fly optimization algorithms and Wan et al. [31] presented a intelligent algorithm which embeds the chaos searching technique into particles warm optimization. In addition, genetic algorithm [32], tabu search approach [33], particle swarm algorithm [34] and ant colony algorithm [35] have been widely developed to solve BLPP.

State transition algorithm (STA) [36] is a recently emerging intelligent optimization algorithm and exhibits good global search ability and fast convergence speed. The STA utilizes the state space expressions in modern control theory as a unified framework for generating candidate solutions, and the specific search operators are designed according to the requirements of the optimization algorithm's globality, optimality, and rapidity. As a new paradigm for solving optimization problems, STA can find the global optimal solution with higher probability and faster speed by designing state transformation operators that can generate geometric neighborhoods with regular shapes and controllable sizes. This paper proposes a nested algorithm called hybrid bi-level STA (HBLSTA) based on STA to solve the BLP models of decentralized supply chain. In terms of nested method, an poor lower-level solution may cause errors in the optimization of the BLPP. However, the convergence rate of STA decreases rapidly in the late iteration. In order to find an accurate lower-level optimal solution, a large number of iterations will be carried out. Simultaneously, gradient-based sequential quadratic programming (SQP) shows greater advantages in accuracy and convergence rate when searching for local optimal solutions. Therefore, the hybrid STA strategy which combines STA and SQP is employed to ensure that the lower-level solution with sufficiently high accuracy is found. Furthermore, each upper-level solution corresponds to a lower-level problem that needs to be solved, so the computational efficiency of the nested algorithm is greatly reduced. The mapping approximation strategy, thence, is applied to improve computational efficiency. Through the verification of numerical experiments, the HBLSTA can obtain a equilibrium solution of the established nonlinear BLP models and effectively coordinate the decision-making schemes of the manufacturer and the distributors.

The contribution of the paper can be summarized in the following two aspects:

1. BLP is utilized to model a decentralized supply chain that includes a manufacturer and multiple distributors. The models not

Table 1
Summary of related definitions and notations.

	Notation	Description
The lower level	$y \in Y$	Decision vectors and space.
	$f : \mathbb{R}^n \times \mathbb{R}^m \to \mathbb{R}$	Objective function.
	$g_j : X \times Y \to \mathbb{R}, j = 1, \ldots, J$	Constraint functions.
	$\Omega(x) = \{y \in Y \mid g(x, y) \leq 0\}$	Feasible region.
	$\Psi(x) = \{y \mid y \in \arg\min\{f(x, y) \mid y \in \Omega(x)\}\}$	Reaction set
The upper level	$x \in X$	Decision vectors and space.
	$F : \mathbb{R}^n \times \mathbb{R}^m \to \mathbb{R}$	Objective function.
	$G_i : X \times Y \to \mathbb{R}, i = 1, \ldots, I$	Constraint functions.
	$M(x) = \{x \in X \mid \exists\ y \text{ such that } (x, y) \in \Omega\}$	Feasible region
BLPP	$\Omega = \{(x, y) \in X \times Y \mid G(x, y) \leq 0, g(x, y) \leq 0\}$	Constraint region
	$\text{FS} = \{(x, y) \mid x \in M(x), y \in \Psi(x)\}$	Feasible set

only consider the coordination of pricing, advertising, ordering, transportation and inventory management, but also restrict the manufacturer's budget at each interval.

2. A bi-level optimization algorithm based on hybrid STA is developed to effectively address the formulated models. The hybrid STA and mapping approximation strategy are utilized to improve the accuracy and computational efficiency of the algorithm.

The remainder of this paper is organized as follows. Section 2 describes a brief introduction of the BLPP. In Section 3, two BLP models (Manufacturer–leader & Distributor–leader) are developed by formulating the manufacturer and the distributor problems. Section 4 proposes a bi-level optimization algorithm based on hybrid STA to solve the BLP models. Section 5 is devoted to experimental results which is used to verify the effectiveness of the proposed algorithm and models. Finally, the conclusions are given in Section 6.

2. Mathematical model of BLPP and related concepts

The BLPP contains two levels optimization tasks that have their own objectives and constraints. Generally, decision vectors, objective functions and constraints of two levels are different and lower optimization task is nested within the upper one. In the context of BLPP, the upper-level decision maker (i.e., the leader) first specifies a programming, after that lower-level decision maker (i.e., the follower) develops the optimal response strategy according to full knowledge of the leader's action. The general formulation for BLPP can be written as follows:

$$\begin{aligned} \min_{x \in X, y \in Y} \ & F(x, y) \\ \text{s.t. } & G_i(x, y) \leq 0, i = 1, \ldots, I \\ & y \in arg \min_{y \in Y} \ f(x, y) \\ & \quad \text{s.t. } g_j(x, y) \leq 0, j = 1, \ldots, J \end{aligned} \tag{1}$$

where $x \in \mathbb{R}^n$, F and G_i represent decision vectors, objective functions and constraints of the upper level, respectively. Meanwhile, $y \in \mathbb{R}^m$, f and g_j indicate decision vectors, objective functions and constraints of the lower level problem respectively. The BLPP is a NBLPP if one of the objective functions or constraints is nonlinear.

In Eq. (1), the lower-level problem can be regarded as a special constraint condition of the upper-level problem. For the upper-level problem, the optimization task is executed with respect to upper level decision variables x, and the lower-level variables y exist as parameters. The lower-level variables as parameters need to be obtained by solving the lower-level optimization problem. The upper level decision vectors are treated as parameters while optimizing the lower-level problem, and then the obtained optimal lower level solution is transferred to the upper level.

Some related definitions and notations are briefly introduced in Table 1. Ω denotes the constraint region of BLPP, and $\Omega(x)$ represents the feasible region of the lower level for each fixed $x \in X$. For every given x, the follower's rational reaction set is shown as $\Psi(x)$. $M(x)$ is the mapping of Ω onto the decision space of the upper level when the variables of the lower level are fixed. Finally, the feasible set of BLPP is defined as FS.

3. BLPP in decentralized supply chain

3.1. Problem description

The object of this investigation is the decentralized supply chain which contains one manufacturer and multiple distributors. Manufacturers produce products and wholesale them to various distributors, and then distributors provide products to different markets whose sizes, geographic locations, selling prices, and the effects of advertising on prices are different. The manufacturer decides about the wholesale price, production (replenishment) interval and the allocation of distributors' demands. The distributor, meanwhile, determines the selling price and advertising expenditure in each market. In the framework of Stackelberg game theory, the decisions of manufacturer and distributors are scattered and mutually interdependent, and their leader and follower status can be transformed during the decision-making process. The general decision-making process of the decentralized supply chain is shown in Fig. 1.

In the proposed BLP models, the leader can obtain all the information about the follower, so as to predict the optimal decision of the follower and makes the policies that maximize the profits. The models are utilized to find a coordinated decision-making strategy between the manufacturer and distributors. When the manufacturer is the leader, the upper-level manufacturer formulates the production and transportation plans by predicting the optimal response of the distributors, and every distributor determines the price and advertising expenditures based on the upper-level interactive information to maximize their profits. The distributors' programming also have an impact on the manufacturer. When the distributor is the leader the process is similar, and the difference is that the distributors are fully aware of the manufacturer's optimal decision. Additionally, in order to better describe the demands in different markets, we assume that the demand function is a joint nonlinear function of selling price and advertising costs. At the same time, the constraints of inventory capacity, production capacity and budget are considered.

3.2. Notation and model assumption

Table 2 is used to represent the notations employed in BLP models. The models developed in this paper are based on the following assumptions:

1. There are several different markets, and the demands in these markets are jointly affected by advertising expenditures and selling price.

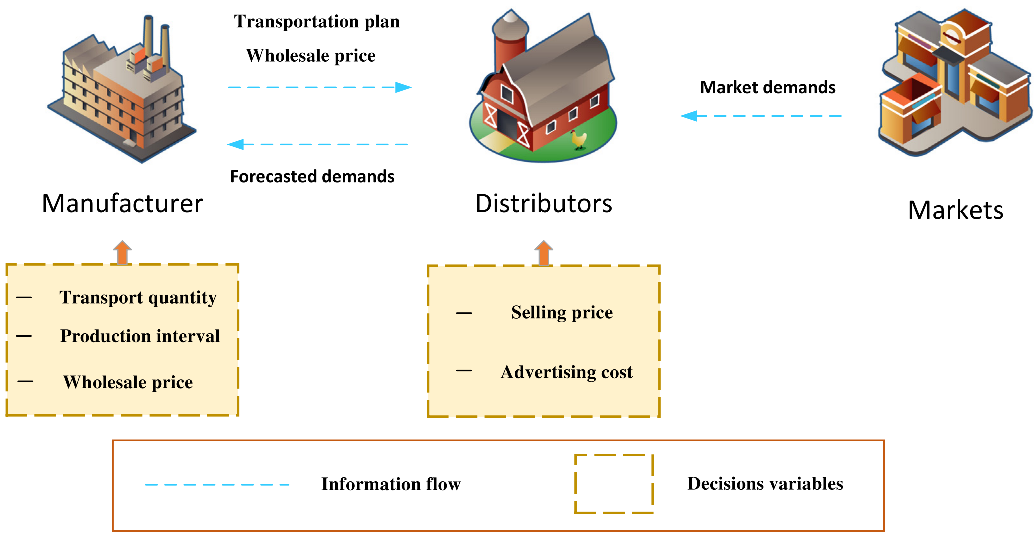

Fig. 1. The decision-making process of the decentralized supply chain.

Table 2
Notations for BLP models.

Notations	Definitions
Parameters (Manufacturer)	
Pc	Production cost ($/Unit)
Sc	Setup cost ($/Setup)
Mh	Percentage of inventory holding cost ($/Unit)
G	Maximum production capacity (Unit)
Tc_j	Unit transportation cost from manufacturer to distributor j ($/Unit)
B_s	Maximum production budget in each interval ($)
Parameters (Distributor)	
Oc_j	Ordering cost for distributor j ($/Order)
Dh_j	Percentage of inventory holding cost for distributor j ($/Order)
C_j	Inventory capacity of distributor j (Unit)
A	Advertising budget limit ($)
D_o	Products produced in each interval (Unit)
Parameters (Markets)	
b_k	Demand scale parameter of market k (Unit)
α_k	Price elasticity of market k (1/$)
β_k	Advertising expenditure elasticity of market k (1/$)
Decision variables (Manufacturer)	
p_m	Wholesale price ($/Unit)
T	Production (replenishment) interval
s_j	Quantity of products shipped from manufacturer to distributor j (Unit)
Decision variables (Distributor)	
p_d	Selling price ($)
a_k	Advertising expenditure in market k ($)

2. Production and replenishment have the same interval, and each production (replenishment) interval is equal. Besides, no shortage is allowed.
3. Under the premise of maximizing profits, the manufacturer and distributors only consider production resources within the supply chain.
4. The capital budget of manufacturer in each production (replenishment) interval is limited.
5. The transportation costs of distributors to markets are included in the selling price.

3.3. Bi-level programming models for decentralized supply chain

Based on the above assumptions and notations, a two-level decentralized supply chain with one manufacturer and multiple distributors is considered. The manufacturer wholesales the products to distributors at a wholesale unit price (p_m). Then, distributors sell goods to different markets at unit selling price (p_d). At the same time, the demand of each market ($D_k(p_d, a_k)$) is a joint nonlinear function of selling price and advertising expenditure as follows:

$$D_k(p_d, a_k) = b_k \cdot p_d^{-\alpha_k} \cdot a_k^{\beta_k} \tag{2}$$

where, price elasticity ($\alpha_k > 1$) and advertising expenditure ($0 < \beta_k < 1$, $\beta_k+1 < \alpha_k$) elasticity represent the impact of selling price and advertising expenditure on demand, respectively.

The incomes of the manufacturer and distributors in the whole planning horizon are respectively denoted by I_m and I_d as follows:

$$I_m = p_m \cdot \sum_{k=1}^{K} b_k \cdot p_d^{-\alpha_k} \cdot a_k^{\beta_k} \tag{3}$$

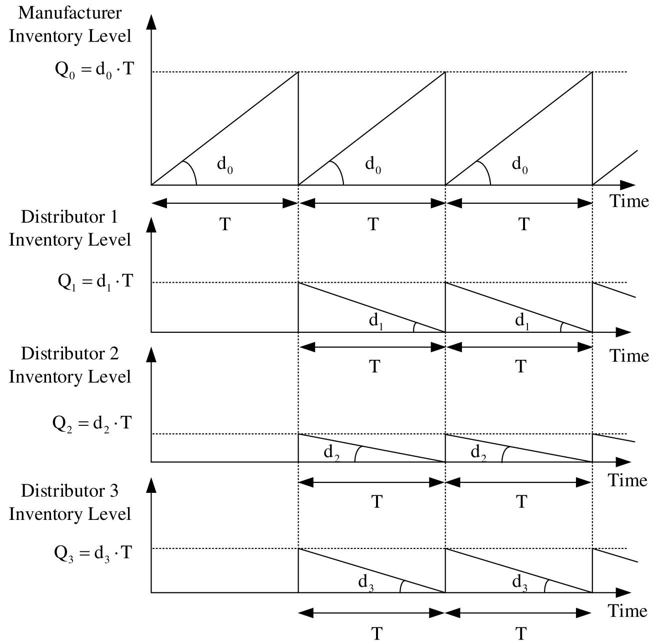

Fig. 2. Inventory levels of the manufacturer and distributors.

$$I_d = p_d \cdot \sum_{k=1}^{K} b_k \cdot p_d^{-\alpha_k} \cdot a_k^{\beta_k} \tag{4}$$

The raw material cost of the manufacturer and the wholesale cost of the distributors are denoted by W_m and W_d respectively, we have:

$$W_m = Pc \cdot \sum_{k=1}^{K} b_k \cdot p_d^{-\alpha_k} \cdot a_k^{\beta_k} \tag{5}$$

$$W_d = p_m \cdot \sum_{k=1}^{K} b_k \cdot p_d^{-\alpha_k} \cdot a_k^{\beta_k} \tag{6}$$

When the production interval (replenishment interval) is T, the quantity of products produced in each production interval can be obtained as $Q_0 = d_0 \cdot T$, where d_0 indicates the total demand of the markets. Products produced in a interval are delivered to distributor j whose total demand in the planning horizon is d_j, and distributor j, therefore, needs to replenish $Q_j = d_j \cdot Q_0/D_0 = d_j \cdot T$ unit of goods in a interval. Taking three distributors as an example, the inventory levels of the manufacturer and distributors are shown in Fig. 2.

According to Fig. 2, it can be seen that the average inventory levels of the manufacturer and distributors in a interval are $T/2 \cdot \sum_{k=1}^{K} b_k \cdot p_d^{-\alpha_k} \cdot a_k^{\beta_k}$ and $T/2 \cdot \sum_{j=1}^{J} s_j$, respectively. Consequently, we can get the inventory costs of the manufacturer and distributors which are denoted by H_m and H_d respectively as follows:

$$H_m = T/2 \cdot Mh \cdot Pc \cdot \sum_{k=1}^{K} b_k \cdot p_d^{-\alpha_k} \cdot a_k^{\beta_k} \tag{7}$$

$$H_d = p_m \cdot T/2 \cdot \sum_{j=1}^{J} s_j \cdot Dh_j \tag{8}$$

The manufacturer's total setup cost and the distributors' total order cost are denoted by C_m and C_d respectively:

$$C_m = 1/T \cdot Sc \tag{9}$$

$$C_d = 1/T \cdot \sum_{j=1}^{J} Oc_j \tag{10}$$

The advertising expenditure of distributors is represented by A_d as follows:

$$A_r = \sum_{k=1}^{K} a_k \tag{11}$$

In addition, since the distributors' transportation costs to markets are included in the selling price, only the manufacturer's transportation costs need to be formulated as follows:

$$T_m = \sum_{j=1}^{J} Tc_j \cdot s_j \tag{12}$$

According to Eqs. (3)–(12), the net profits of the manufacturer and distributors are formulated as follows:

$$\begin{aligned} P_M &= I_m - W_m - C_m - H_m - T_m \\ &= (p_m - Pc) \cdot \sum_{k=1}^{K} b_k \cdot p_d^{-\alpha_k} \cdot a_k^{\beta_k} - \frac{1}{T} \cdot Sc \\ &\quad - \frac{T}{2} \cdot Mh \cdot Pc \cdot \sum_{k=1}^{K} b_k \cdot p_d^{-\alpha_k} \cdot a_k^{\beta_k} - \sum_{j=1}^{J} Tc_j \cdot s_j \end{aligned} \tag{13}$$

$$\begin{aligned} P_D &= I_d - W_d - C_d - H_d - A_d \\ &= (p_d - p_m) \cdot \sum_{k=1}^{K} b_k \cdot p_d^{-\alpha_k} \cdot a_k^{\beta_k} - \frac{1}{T} \cdot \sum_{j=1}^{J} Oc_j \\ &\quad - p_m \cdot \frac{T}{2} \cdot \sum_{j=1}^{J} s_j \cdot Dh_j - \sum_{k=1}^{K} a_k \end{aligned} \tag{14}$$

In the manufacturer's planning level, the objective function is the maximum net profit P_M. The constraints are provided as follows: (1) The total demand of markets cannot exceed the maximum production capacity of manufacturer $\sum_{k=1}^{K} b_k \cdot p_d^{-\alpha_k} \cdot a_k^{\beta_k} \leq G$. (2) In each interval, the production budget cannot be exceeded $T \cdot Pc \cdot \sum_{k=1}^{K} b_k \cdot p_d^{-\alpha_k} \cdot a_k^{\beta_k} \leq B_s$. (3) The quantity of goods shipped from the manufacturer to the distributors should meet the capacity of the distributor's depots $0 \leq s_j \leq C_j, j = 1, 2, \ldots, J$. (4) The wholesale price is greater than the production price $p_m \geq Pc \cdot (1 + \delta_M)$, where δ_M is a scaling constant. (5) Production (replenishment) interval meets $0 < T \leq 1$.

On the other hand, the distributors also aim to maximize the net profit. The constraints are formulated as follows: (1) The inventory capacity of distributors can meet the total demand of the markets $\sum_{k=1}^{K} b_k \cdot p_d^{-\alpha_k} \cdot a_k^{\beta_k} \leq \sum_{j=1}^{J} C_j$. (2) The total number of goods purchased by distributors completely meets the demand of every market $\sum_{j=1}^{J} s_j = \sum_{k=1}^{K} b_k \cdot p_d^{-\alpha_k} \cdot a_k^{\beta_k}$. (3) The selling price is greater than the wholesale price $p_d \geq p_m \cdot (1 + \delta_D)$, where δ_D is a scaling constant. (4) Expenses used for advertising must be within budget limits $0 \leq a_k \leq A, k = 1, 2, \ldots, K$.

When developing the BLP models from the perspective of manufacturer, the main consideration is to get more profits in the operation of the supply chain, so the manufacturer is appointed as the leader and the distributors are the follower. From the perspective of distributors, instead, the distributors are designated as the leader and the manufacturer is the follower. In order to obtain the maximum profits under the constraints, they determine their decision variables separately and formulate appropriate coordination strategies within the planning horizon of supply chain.

3.3.1. The manufacturer–leader model

The Manufacturer–leader (ML) model assumes that manufacturer holds the manipulative power and acts as the leader.

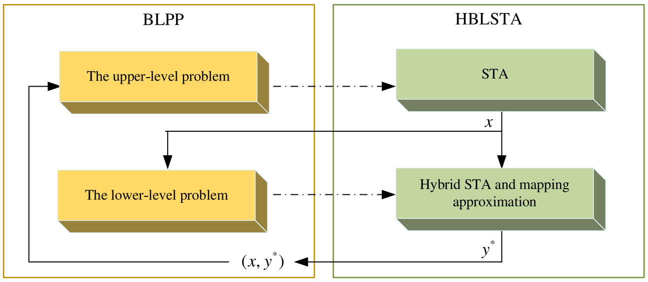

Fig. 3. A brief framework of the HBLSTA.

According to the above mentioned analysis, the ML model can be expressed as follows:

$$\max_{p_m,T,s_j} P_M = (p_m - Pc)\cdot\sum_{k=1}^{K} b_k\cdot p_d^{-\alpha_k}\cdot a_k^{\beta_k} - \frac{1}{T}\cdot Sc - \frac{T}{2}\cdot Mh\cdot Pc\cdot\sum_{k=1}^{K} b_k\cdot p_d^{-\alpha_k}\cdot a_k^{\beta_k} - \sum_{j=1}^{J} Tc_j\cdot s_j$$

Subject to

$$\sum_{k=1}^{K} b_k\cdot p_d^{-\alpha_k}\cdot a_k^{\beta_k} \le G$$
$$T\cdot Pc\cdot\sum_{k=1}^{K} b_k\cdot p_d^{-\alpha_k}\cdot a_k^{\beta_k} \le B_s$$
$$0 \le s_j \le C_j, j = 1,2,\ldots,J$$
$$p_m \ge Pc\cdot(1+\delta_M)$$
$$0 < T \le 1$$

where p_d, a_k are solved by the following problem

$$\max_{p_d,a_k} P_D = (p_d - p_m)\cdot\sum_{k=1}^{K} b_k\cdot p_d^{-\alpha_k}\cdot a_k^{\beta_k} - \frac{1}{T}\cdot\sum_{j=1}^{J} Oc_j - p_m\cdot\frac{T}{2}\cdot\sum_{j=1}^{J} s_j\cdot Dh_j - \sum_{k=1}^{K} a_k$$

Subject to

$$\sum_{k=1}^{K} b_k\cdot p_d^{-\alpha_k}\cdot a_k^{\beta_k} \le \sum_{j=1}^{J} C_j$$
$$\sum_{j=1}^{J} s_j = \sum_{k=1}^{K} b_k\cdot p_d^{-\alpha_k}\cdot a_k^{\beta_k}$$
$$p_d \ge p_m\cdot(1+\delta_D)$$
$$0 \le a_k \le A, k = 1,2,\ldots,K \tag{15}$$

According to Eq. (15), the upper-level manufacturer, as the leader, makes the decision first. Under the production constraints, the manufacturer determines wholesale price (p_m), production (replenishment) interval (T) and transportation strategy (s_j). The distributers are the follower, who makes decision about selling prices (p_d) and advertising expenditures (a_k) according to the possible strategy of the manufacturer. Next, The roles of manufacturer and distributors in the planning will be switched.

3.3.2. The distributer–leader model

Here we consider the distributers as the leader and the manufacturer as the follower. Similar to the ML model, the Distributer–leader (DL) model can be formulated as follows:

$$\max_{p_d,a_k} P_D = (p_d - p_m)\cdot\sum_{k=1}^{K} b_k\cdot p_d^{-\alpha_k}\cdot a_k^{\beta_k} - \frac{1}{T}\cdot\sum_{j=1}^{J} Oc_j - p_m\cdot\frac{T}{2}\cdot\sum_{j=1}^{J} s_j\cdot Dh_j - \sum_{k=1}^{K} a_k$$

Subject to

$$\sum_{k=1}^{K} b_k\cdot p_d^{-\alpha_k}\cdot a_k^{\beta_k} \le \sum_{j=1}^{J} C_j$$
$$\sum_{j=1}^{J} s_j = \sum_{k=1}^{K} b_k\cdot p_d^{-\alpha_k}\cdot a_k^{\beta_k}$$
$$p_d \ge p_m\cdot(1+\delta_D)$$
$$0 \le a_k \le A, k = 1,2,\ldots,K$$

where p_m, T, s_j are solved by the following problem

$$\max_{p_m,T,s_j} P_M = (p_m - Pc)\cdot\sum_{k=1}^{K} b_k\cdot p_d^{-\alpha_k}\cdot a_k^{\beta_k} - \frac{1}{T}\cdot Sc - \frac{T}{2}\cdot Mh\cdot Pc\cdot\sum_{k=1}^{K} b_k\cdot p_d^{-\alpha_k}\cdot a_k^{\beta_k} - \sum_{j=1}^{J} Tc_j\cdot s_j$$

Subject to

$$\sum_{k=1}^{K} b_k\cdot p_d^{-\alpha_k}\cdot a_k^{\beta_k} \le G$$
$$T\cdot Pc\cdot\sum_{k=1}^{K} b_k\cdot p_d^{-\alpha_k}\cdot a_k^{\beta_k} \le B_s$$
$$0 \le s_j \le C_j, j = 1,2,\ldots,J$$
$$p_m \ge Pc\cdot(1+\delta_M)$$
$$0 < T \le 1 \tag{16}$$

The ML model and DL model are both NBLPPs which are NP-hard, so a hybrid bi-level state transition algorithm will be developed in the following section to tackle these problems.

4. The proposed HBLSTA

In consideration of the hierarchical structure characteristics of the NBLPP, a novel algorithm called HBLSTA with nested structure is constructed to tackle ML and DL problems. The proposed algorithm solves the BLPP by simulating the sequential decision-making procedure of BLPP. HBLSTA contains two levels, where upper and lower level algorithms can exchange information on the optimal solutions to the different levels. The STA with powerful global search capability is utilized in upper level to solve the leader's problem based on the prediction of the follower's response, and a hybrid STA strategy combining STA and SQP is developed in lower level to solve the follower's problem and get the optimal response for the given set of leader's decision variables. In addition, in order to improve the computational efficiency, a mapping approximation strategy is used to reduce the number of calls to the lower-level optimizer.

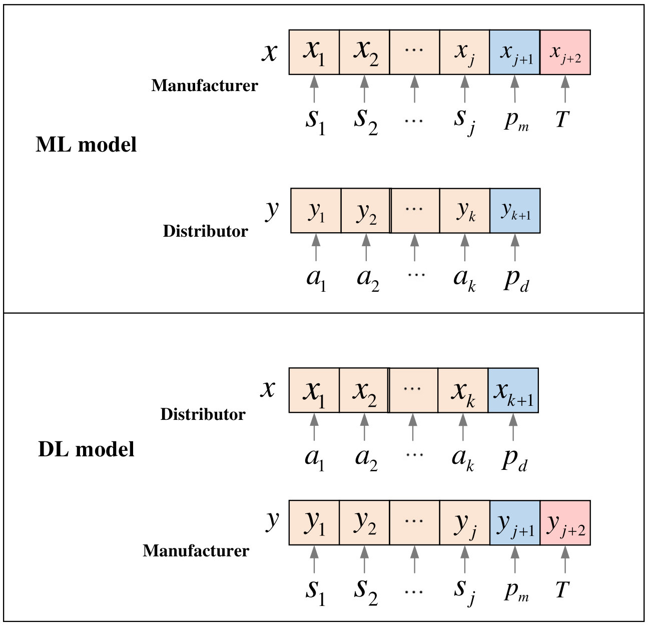

Fig. 4. Solution encoding of the proposed approach.

Fig. 3 depicts a brief framework of the HBLSTA. The upper-level STA starts with generating an initial solution (x) for the leader's problem and the solution is then taken by the lower-level problem. The follower's optimal response (y^*) is obtained through the combination of hybrid STA and mapping approximation strategies and then returned to the upper level. After evaluating the initial solution for the leader's problem (x, y^*), the first iteration is complete. The process repeats in each iteration until the stop condition is met or the optimal solution for the upper-level problem (x^*, y^*) is obtained.

4.1. Solution encoding

The decision variables of the manufacturer and the distributor problems can be coded as an array corresponding to the upper and lower level solutions in the algorithm, respectively. In the ML model, the manufacturer's decision variables p_m, T and s_j can be coded as an array x corresponding to the upper-level solution and p_d and a_k of the distributor corresponds to the lower-level solution y (see Fig. 4). Similarly, the encoding of the solution in the DL model can be obtained.

4.2. Hybrid STA strategy

The STA is an intelligent random global optimization algorithm that first proposed by Zhou in 2012. The STA grasps the essence of the optimization algorithm, and constructed a framework for solving optimization problems based on structuralism learning. Because of the powerful global search capability, simple implementation and few parameters, the STA has shown its superior performance in many practical applications, such as nonlinear system identification [37], industrial process optimization [38,39], machine learning [40,41] and other fields [42,43]. However, the research on STA has not yet involved the solution of NBLPP.

The basic idea of the STA is to treat a solution as a state, and the generation and update of the solution as a state transition process. The formula of the framework is as follows:

$$\begin{cases} x_{k+1} = A_k \cdot x_k + B_k \cdot u_k \\ y_{k+1} = f(x_{k+1}) \end{cases} \tag{17}$$

where, the $x_k \in \mathbb{R}^n$ is the current state. A_k and B_k are state transition matrices which can be seen as an operator in the optimization algorithm. u_k represents the information provided by the current state and the historical state, and $f(\cdot)$ is the objective function.

The state transformation operators are designed based on above framework. Especially, there are four basic operators for local and global search.

(1) Rotation transformation

$$x_{k+1} = x_k + \alpha \cdot \frac{1}{n \cdot \|x_k\|_2} \cdot R_r \cdot x_k \tag{18}$$

where $\alpha > 0$ is the rotation factor, $R_r \in \mathbb{R}^{n\times n}$ indicates a random matrix whose elements are uniformly distributed between $[-1, 1]$, and $\|\cdot\|_2$ is the L2-norm of a vector.

(2) Translation transformation

$$x_{k+1} = x_k + \beta \cdot R_t \cdot \frac{x_k - x_{k-1}}{\|x_k - x_{k-1}\|_2} \tag{19}$$

where $\beta > 0$ is the translation factor, $R_t \in \mathbb{R}$ represents a random variable uniformly distributed between $[0, 1]$.

(3) Expansion transformation

$$x_{k+1} = x_k + \gamma \cdot R_e \cdot x_k \tag{20}$$

where $\gamma > 0$ is called the expansion factor, $R_e \in \mathbb{R}^{n\times n}$ is a random diagonal matrix whose elements follow a Gaussian distribution.

(4) Axesion transformation

$$x_{k+1} = x_k + \delta \cdot R_a \cdot x_k \tag{21}$$

where $\delta > 0$ is called the axesion factor, $R_a \in \mathbb{R}^{n\times n}$ is a random diagonal matrix whose nonzero elements follow a Gaussian distribution.

Using STA to address NBLPP in a nested framework, one of the key issues is that the convergence rate of the algorithm declines rapidly as the iteration progresses. Moreover, it is difficult to balance global search and local search in the late iteration, which needs to be avoided in the optimization of the lower-level problem. Meanwhile, gradient-based algorithms converge quickly in local search. When solving the optimization problems of the upper and lower levels, a hybrid STA strategy is introduced in order to speed up the convergence on the premise of ensuring accuracy. The hybrid STA is divided into two stages: First, STA is utilized to find a rough global solution region. Afterwards, a gradient-based method SQP will be adopted for the purpose of converging faster to a solution with sufficiently high accuracy in the found region.

4.3. Mapping approximation method

According to the definitions of the BLPP in Table 1. $\Psi(x)$ can be understood as the mapping of the upper-level decision and the corresponding lower-level optimal solution. If the mathematical formula of Ψ-mapping can be determined, the optimal lower-level solution for a given upper-level solution can be obtained directly without solving the lower-level problem. So that computational efficiency will be greatly improved.

However, Ψ-mapping is difficult to determine directly, so an iterative strategy is applied to approximate it. Assuming $\mathcal{H}$ is the hypothesis space. The hypothesis space contains all the functions that can be used to generate the mapping between the upper-level decision vector and the optimal lower-level decision vector. For a given sample consisting of N upper-level points (x) and corresponding optimal lower-level points (y^*), a mapping $\hat{\psi} \in \mathcal{H}$ is obtained by minimizing the empirical error on the sample as follows:

$$\hat{\psi} = \arg\min_{h\in\mathcal{H}} \sum_{n\in N} L(h(x_n), y_n^*) \tag{22}$$

where $L : \mathbb{R}^m \times \mathbb{R}^m \rightarrow \mathbb{R}$ represents the prediction error, x_n is any given upper-level decision, and y_n^* is corresponding optimal lower-level solution. The formulation of the prediction error is as follows :

$$L(h(x_n), y_n^*) = |y_n^* - h(x_n)|^2 \tag{23}$$

It is required too many sample points to directly approximate Ψ-mapping. Therefore, a local mapping (ψ_{loc}) is created for an upper-level solution ($\tilde{x}$) that needs to obtain the lower-level optimal solution through the mapping relationship.

In order to create ψ_{loc}-mapping, it is necessary to use hybrid STA to solve the lower-level problems corresponding to some upper-level members to provide samples. Then, the members whose Euclidean distance is shortest to $\tilde{x}$ are selected from the samples to get ψ_{loc}-mapping. In addition, the hypothesis space is restricted to be composed of second-order polynomials, and thence at least $n(n+1)/2$ points are required to approximate the ψ_{loc}-mapping whose upper-level vector is n-dimensional. Considering the performance of the ψ_{loc}-mapping, we use $n(n+1)$ samples for the approximation.

4.4. Constraints handling and fitness assignment

A BLPP contains two levels of optimization tasks, and there may be constraints on both levels. HBLSTA uses similar constraint handling schemes to process the upper and lower constraints. The main idea is to assigns the feasible solution a higher fitness value than infeasible one [44]. For the two upper and lower level solution pairs (x_i, y_i) and (x_j, y_j) of the BLPP, the fitness allocation scheme is as follows:

(1) If both (x_i, y_i) and (x_j, y_j) are feasible solutions that can meet the constraints, the fitness value depends on their corresponding function value.
(2) If one solution of (x_i, y_i) and (x_j, y_j) is feasible and the other is infeasible, a higher fitness value is given to the feasible one.
(3) If (x_i, y_i) and (x_j, y_j) are infeasible, the fitness value looks at the overall constraint violation which is the sum of all equality and inequality constraint violations.

4.5. The HBLSTA for solving BLPP

According to the framework of HBLSTA, the algorithm has a similar structure to BLPP. The HBLSTA contains the upper and lower STAs, which are respectively applied to the optimization tasks of the upper and lower levels in BLPP. In addition, considering that each upper-level solution corresponds to a lower-level problem, solving all of these optimization problems will greatly reduce the computational efficiency. Therefore, the mapping approximation strategy is introduced into the lower-level optimization task to avoid frequently solving of the lower-level problems. The flowchart of HBLSTA is shown in Fig. 5.

The overall approach of HBLSTA is shown in Algorithm 1. Using an interactive iteration mechanism, the information of the upper and lower solutions is constantly updated, and HBLSTA will find the equilibrium optimal solution of BLPP. The lower-level optimal solution is obtained by mapping approximation and hybrid STA, and the specific process of the lower-level hybrid STA is illustrated in Algorithm 2.

In Algorithm 1, when calculating the fitness value of $z_1, \ldots, z_N$, the upper-level objective function is considered as the fitness function and the fitness assignment method is in Section 4.4. The termination criterion is that *iteration* reaches the Max_{It} or fitness value no longer improves. In Algorithm 2, the lower-level objective function is considered as the fitness function and the termination criterion is the same as Algorithm 1.

Algorithm 1 The overall approach of HBLSTA.

Input:
The parameters of STA: α, β, γ, δ and SE;
Maximum iteration Max_{It};
Output:
Optimal solution;
Generate N initial feasible upper-level solutions $x_1, \ldots, x_N$;
For the given upper-level solution x_i $(i = 1, \ldots, N)$, solve the corresponding lower-level optimization problem with algorithm 2 and obtain the lower-level optimal solution y_i^*;
Calculate the fitness value of the initial solutions $z_1, \ldots, z_N$ $(z_i = (x_i, y_i^*))$;
Save $z_1, \ldots, z_N$ in the archive $\mathcal{A}$;
Choose the solution with the largest fitness value from $z_1, \ldots, z_N$ as the initial optimal solution z^*;
iteration = 0;
while The termination criterion is not met **do**
  Generate SE candidate states $x_1, \ldots, x_{SE}$ by the state transformation operators;
  For each candidate state x_k $(k = 1, \ldots, SE)$, obtain the corresponding optimal lower-level solutions y_k^* through hybrid STA optimization or mapping approximation;
  if the mapping approximation condition is met **then**
    Utilize the archive $\mathcal{A}$ to construct a local mapping relationship $y = \psi_{loc}(x)$ and directly obtain y_k^*;
  else
    Solve the corresponding lower-level optimization problem with algorithm 2 and obtain the lower-level optimal solution y_k^*;
    Add $z_k = (x_k, y_k^*)$ to the archive $\mathcal{A}$;
  end if
  Calculate the fitness value of the candidate state corresponding solutions $z_1, \ldots, z_{SE}$;
  Update the optimal solution z^* according to the fitness value;
  $iteration = iteration + 1$;
end while
return optimal solution;

Algorithm 2 The lower level hybrid STA of HBLSTA.

Input:
Initialize parameters of STA: α, β, γ, δ and SE;
Maximum iteration Max_{It};
upper-level solution x_i
Output:
Lower-level optimal solution;
Generate initial feasible lower-level solution y_0;
Calculate the fitness value of y_0;
Set y_0 as the initial lower-level optimal solution y^*;
while The termination criterion is not met **do**
  Generate SE candidate states $y_1, \ldots, y_{SE}$ by the state transformation operators;
  Calculate the fitness values of $y_1, \ldots, y_{SE}$;
  Update the lower-level optimal solution y^* according to the fitness value;
  $iteration = iteration + 1$;
end while
return lower-level optimal solution;

5. Computational experiments and discussion

This section contains two parts: algorithm performance experiments and model evaluation. A set of benchmark BLPPs are employed to test the performance of the HBLSTA and then HBLSTA is utilized to address the proposed BLP models. All experiments are implemented in MATLAB R2019b with an AMD R9-4900H, 3.30 GHz processor and 8 GB of RAM on a 64-bit Windows 10 operating system.

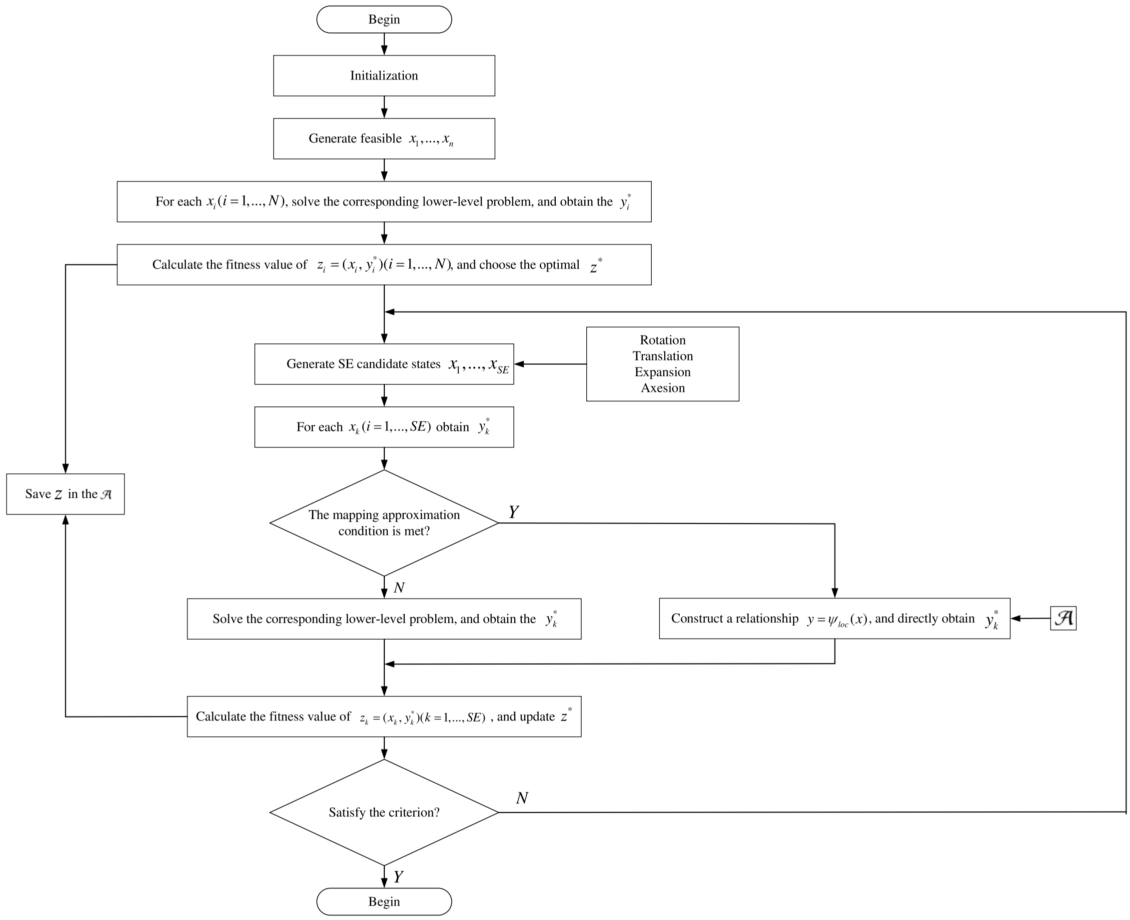

Fig. 5. The flowchart of HBLSTA.

Table 3
Parameter setup.

Parameter	SE	α	β	γ	δ
Value	8	1	1	1	1

5.1. Algorithm performance experiments

The parameter setting of HBLSTA [36] is shown in Table 3. The TP test suite [45] which contains 10 benchmark instances from TP1 to TP10 are tested to evaluate the performance of HBLSTA. For better comparison, the relevant properties of problems in the TP test suite are shown in Table 4.

In the experiments, the accuracy and efficiency are utilized as the primary performance indicator. In terms of accuracy, the absolute difference between the best known objective value and the obtained objective value is measured. That is, $Acc^u = |F - F^*|$ and $Acc^l = |f - f^*|$, where F^* and f^* are the optimal function values of the upper-level and lower-level respectively, and F and f are respectively the upper-level and lower-level function values obtained by a certain algorithm. In terms of efficiency, the numbers of function evaluations (denoted by UFEs at the upper level and LFEs at the lower level) are reported in the experimental results to compare the computational resources consumed by different algorithms. Three algorithms including the nested bilevel evolutionary algorithm (NBLEA) [46], bilevel evolutionary algorithm based on quadratic approximations (BLEAQ) [45] and bilevel evolutionary algorithm based on quadratic approximations 2 (BLEAQ2) [47] are considered to illustrate the performance of HBLSTA and for each test problem. NBLEA is a classic nested algorithm which solves the upper-level and lower-level problems with genetic algorithms. BLEAQ adds strategies such as iterative local search on the basis of NBLEA to improve the efficiency, and some lower-level optimal solutions can be obtained directly by quadratic approximating. BLEAQ2 introduces the optimal value mapping into BLEAQ for the case of multiple optimal solutions in the lower level. They are currently more advanced algorithms for solving NBLPPs. All algorithms are independently run 30 times and record the Acc^u, Acc^l, UFEs and LFEs. Furthermore, the median of various indicators is used as the main basis for comparison while the best, worst and mean of various indicators are also utilized to further verify the performance of the algorithms. The experimental results are given in Table 5, Table 6, Tables 7 and 8, respectively.

Tables 5 and 6 show the comparison results of the accuracy of HBLSTA and other algorithms on different benchmark instances. The best result of each test example is marked in bold, and because BLPPs always prioritize the interests of the leader, only the upper-level objective function value is considered when marking. From the results, it can be seen that HBLSTA shows the best performance on all test problems except TP3, but even on TP3, HBLSTA behaves quite competitively. The performance of HBLSTA on TP3 differs very little from the best performing BLEAQ and is much more accurate than other two comparison algorithms. In addition, note that on the two test problems of TP2 and TP8, although other algorithms can achieve the same accuracy as HBLSTA in the upper level, other algorithms will often fall into the local optimal situation in the optimization of the lower level. Due to the hybrid STA strategy which combines the global search capability of STA with the local search capability of

Table 4
The relevant properties of problems in the TP test suite.

Test problem	Upper-level problem			Lower-level problem		
	Dimension	Objective function	Constraint	Dimension	Objective function	Constraint
TP1	2	Non-linear, differentiable, continuous	With linear constraints	2	Non-linear, differentiable, continuous	Unconstrained
TP2	2	Linear, differentiable, continuous	With linear constraints	2	Non-linear, differentiable, continuous	With linear constraints
TP3	2	Non-linear, differentiable, continuous	With non-linear constraints	2	Non-linear, differentiable, continuous	With non-linear constraints
TP4	2	Linear, differentiable, continuous	Unconstrained	3	Linear, differentiable, continuous	With linear constraints
TP5	2	Non-linear, differentiable, continuous	Unconstrained	2	Non-linear, differentiable, continuous	With linear constraints
TP6	1	Non-linear, differentiable, continuous	Unconstrained	2	Non-linear, differentiable, continuous	With non-linear constraints
TP7	2	Non-linear, differentiable, discontinuous	With non-linear constraints	2	Non-linear, differentiable, discontinuous	With linear constraints
TP8	2	Non-linear, non-differentiable, discontinuous	With linear constraints	2	Non-linear, differentiable, continuous	With linear constraints
TP9	10	Non-linear, non-differentiable, discontinuous	Unconstrained	10	Non-linear, differentiable, discontinuous	Unconstrained
TP10	10	Non-linear, non-differentiable, discontinuous	Unconstrained	10	Non-linear, differentiable, discontinuous	Unconstrained

mathematical programming, HBLSTA can improve the accuracy of the solution when solving the lower-level optimization problem. For all test problems, the optimization accuracy of the lower layer of HBLSTA is the highest, which verifies the effectiveness of the hybrid strategy.

The comparison results of the efficiency of different algorithms are shown in Tables 7 and 8. It is generally considered that the calculation resources consumed by the upper and lower level function evaluations are the same, and the best result of each test instance is also marked in bold. The results show that the sum of the UFEs and LFEs of HBLSTA is the smallest on the 10 questions, which indicates that the calculation efficiency of HBLSTA is the highest. The main reason is that the mapping approximation strategy and hybrid STA strategy used by HBLSTA can greatly reduce the evaluation times of the lower level function.

5.2. Model evaluation

In this section, HBLSTA is utilized to solve the proposed BLP models (Eqs. (15) and (16)). In order to evaluate the ability of HBLSTA to find the optimal solutions for the two models, 5 problem samples are defined according to the number of distributors ($J = 2, 3, 4, 5, 6$) and the number of markets ($K = 3, 5, 7, 10, 15$). Furthermore, the parameters in the two models are uniformly distributed. Table 9 shows the domains of the corresponding parameters for manufacturer, distributors, and markets. The parameters in each problem sample are randomly selected from the given domains, and other parameters including G, C_j, B_s and A are directly dependent on the total induced demand.

The parameter setting of HBLSTA is the same as parameters in Table 3. In order to validate the performance of HBLSTA in solving the proposed BLP models, BLEAQ2 is adopted to find near optimal solutions for problem instances. Each problem sample is run independently 30 times, and the average results are shown in Tables 10 and 11, respectively.

Table 5
Accuracy comparison of the HBLSTA with the other three algorithms on the TP test suite (median).

Pr.	Level	HBLSTA	NBLEA	BLEAQ	BLEAQ2
TP1	Acc^u	**2.55e−06**	5.29e−01	2.60e−05	3.63e−06
	Acc^l	**1.00e−06**	9.33e−01	3.34e−05	1.00e−06
TP2	Acc^u	**1.00e−06**	**1.00e−06**	5.00e−00	**1.00e−06**
	Acc^l	**2.13e−05**	**1.00e+02**	1.00e+02	**1.00e+02**
TP3	Acc^u	1.36e−05	8.91e−03	**1.09e−05**	5.25e−04
	Acc^l	2.50e−05	1.40e−03	**2.50e−05**	8.06e−05
TP4	Acc^u	**1.00e−06**	2.58e−02	1.19e−05	1.69e−05
	Acc^l	**1.00e−06**	4.99e−03	1.70e−06	2.03e−06
TP5	Acc^u	**4.70e−06**	5.16e−04	1.90e−04	1.33e−05
	Acc^l	**1.00e−06**	2.07e−03	8.88e−04	1.99e−05
TP6	Acc^u	**1.00e−06**	6.55e−06	6.57e−06	6.57e−06
	Acc^l	**1.00e−06**	3.42e−05	4.03e−06	4.06e−06
TP7	Acc^u	**5.62e−06**	1.62e−03	6.76e−04	6.48e−04
	Acc^l	**5.62e−06**	1.62e−03	6.76e−04	6.48e−04
TP8	Acc^u	**1.00e−06**	1.00e−06	3.23e−06	1.00e−06
	Acc^l	**3.65e−04**	1.00e+02	1.00e+02	1.00e+02
TP9	Acc^u	**1.00e−06**	3.06e−05	5.54e−05	**1.00e−06**
	Acc^l	**1.00e−06**	1.00e−06	1.00e−06	**1.00e−06**
TP10	Acc^u	**1.00e−06**	3.65e−00	3.62e−05	**1.00e−06**
	Acc^l	**1.00e−06**	1.00e−06	1.00e−06	**1.00e−06**

It can be seen from Table 10 that when HBLSTA and BLEAQ2 are used to solve the ML model, the profits of manufacturer and distributors obtained by HBLSTA are higher than that of BLEAQ2. If $J = 2, K = 3$, the profits of manufacturer and distributors obtained by HBLSTA and BLEAQ2 are relatively close.

Table 6
Accuracy comparison of the HBLSTA with the other three algorithms on the TP test suite. (best, worst and mean).

Pr.	Level	HBLSTA			NBLEA			BLEAQ			BLEAQ2		
		Best	Worst	Mean	Best	Worst	Mean	Best	Worst	Mean	Best	Worst	Mean
TP1	Acc^u	**1.00e−06**	**6.13e−06**	**2.42e−06**	4.33e−03	9.87e−01	6.34e−01	3.88e−06	1.87e−01	8.31e−05	1.00e−06	1.77e−04	8.53e−06
	Acc^l	**1.00e−06**	**1.00e−06**	**1.00e−06**	2.62e−02	1.47e−00	2.33e−01	8.71e−06	4.13e−04	1.39e−05	1.00e−06	1.00e−06	1.00e−06
TP2	Acc^u	**1.00e−06**	**1.00e−06**	**1.00e−06**	**1.00e−06**	**1.00e−06**	**1.00e−06**	**1.00e−06**	5.00e−00	2.89e−00	**1.00e−06**	**1.00e−06**	**1.00e−06**
	Acc^l	**6.84e−06**	**1.00e+02**	**1.62e+01**	**4.79e−05**	**1.00e+02**	**4.10e+01**	**8.99e−05**	1.00e+02	8.12e+01	**7.88e−05**	**1.00e+02**	**3.55e+01**
TP3	Acc^u	4.66e−06	5.26e−04	1.93e−05	9.41e−05	7.11e−01	4.23e−03	**3.48e−06**	**5.17e−05**	**1.68e−05**	6.88e−05	7.55e−02	6.81e−04
	Acc^l	1.00e−06	9.83e−05	1.39e−05	3.48e−05	1.21e−01	3.62e−03	**6.40e−06**	**4.41e−04**	**2.89e−05**	9.60e−06	2.13e−04	8.73e−05
TP4	Acc^u	**1.00e−06**	**1.00e−06**	**1.00e−06**	4.57e−05	5.72e−00	3.99e−02	6.17e−06	7.20e−04	1.33e−05	4.59e−06	2.44e−04	1.65e−05
	Acc^l	**1.00e−06**	**1.00e−06**	**1.00e−06**	4.52e−05	8.24e−02	9.63e−03	1.00e−06	4.90e−05	1.25e−06	1.00e−06	6.33e−05	1.10e−06
TP5	Acc^u	**1.63e−06**	**5.19e−05**	**8.72e−06**	1.08e−05	8.32e−03	4.06e−04	2.87e−05	4.03e−02	4.64e−04	2.41e−06	5.08e−04	8.20e−05
	Acc^l	**1.00e−06**	**1.00e−06**	**1.00e−06**	4.09e−04	3.47e−02	6.18e−03	7.02e−06	7.95e−03	6.82e−04	5.08e−06	4.33e−04	2.06e−05
TP6	Acc^u	**1.00e−06**	**1.00e−06**	**1.00e−06**	1.19e−06	8.37e−05	7.09e−06	4.66e−06	9.89e−06	7.63e−06	5.57e−06	6.69e−06	6.58e−06
	Acc^l	**1.00e−06**	**1.00e−06**	**1.00e−06**	8.43e−06	1.22e−04	3.58e−05	1.91e−06	6.14e−06	3.03e−06	1.26e−06	8.93e−06	3.06e−06
TP7	Acc^u	**3.71e−06**	**8.72e−06**	**6.32e−06**	7.56e−05	1.06e−02	3.02e−03	4.33e−05	5.74e−03	4.03e−04	8.44e−05	4.09e−03	4.41e−04
	Acc^l	**1.00e−06**	**4.62e−05**	**5.69e−06**	9.83e−05	7.73e−02	2.01e−03	1.76e−05	4.12e−03	6.16e−04	1.18e−05	3.08e−03	6.17e−04
TP8	Acc^u	**1.00e−06**	**1.00e−06**	**1.00e−06**	**1.00e−06**	**1.00e−06**	**1.00e−06**	2.23e−06	1.23e−05	3.39e−06	**1.00e−06**	2.96e−06	1.28e−06
	Acc^l	**1.45e−04**	**8.64e−04**	**4.05e−04**	**7.04e−03**	**1.00e+02**	**6.23e+01**	8.08e−04	1.00e+02	5.11e+01	**6.49e−02**	1.00e+02	4.01e+01
TP9	Acc^u	**1.00e−06**	**1.00e−06**	**1.00e−06**	9.10e−06	2.06e−04	4.16e−05	3.04e−06	1.00e−03	5.81e−05	**1.00e−06**	**1.00e−06**	**1.00e−06**
	Acc^l	**1.00e−06**	**1.00e−06**	**1.00e−06**	1.00e−06	1.00e−06	1.00e−06	1.00e−06	1.00e−06	1.00e−06	**1.00e−06**	**1.00e−06**	**1.00e−06**
TP10	Acc^u	**1.00e−06**	**1.00e−06**	**1.00e−06**	4.11e−01	6.07e−00	3.48e−00	3.67e−06	8.99e−05	3.67e−05	**1.00e−06**	**1.00e−06**	**1.00e−06**
	Acc^l	**1.00e−06**	**1.00e−06**	**1.00e−06**	1.00e−06	1.00e−06	1.00e−06	1.00e−06	1.00e−06	1.00e−06	**1.00e−06**	**1.00e−06**	**1.00e−06**

Table 7
Efficiency comparison of the HBLSTA with the other three algorithms on the TP test suite (median).

Pr.	Level	HBLSTA	NBLEA	BLEAQ	BLEAQ2
TP1	UFEs	**1.07e+02**	1.26e+03	7.80e+ 02	1.13e+02
	LFEs	**5.51e+02**	1.61e+05	1.50e+ 04	5.75e+02
TP2	UFEs	**1.11e+02**	3.37e+03	1.43e+03	2.46e+02
	LFEs	**5.95e+02**	2.43e+05	1.45e+04	1.77e+03
TP3	UFEs	**1.04e+02**	1.48e+03	3.62e+02	1.52e+02
	LFEs	**5.22e+02**	1.21e+05	4.48e+03	5.30e+02
TP4	UFEs	**3.01e+02**	1.76e+03	2.76e+02	2.93e+02
	LFEs	**2.72e+03**	2.73e+05	1.53e+04	3.31e+03
TP5	UFEs	**2.82e+02**	3.09e+03	1.30e+03	3.19e+02
	LFEs	**1.74e+03**	1.48e+05	1.57e+04	2.43e+03
TP6	UFEs	**1.41e+02**	3.28e+02	2.84e+02	1.89e+02
	LFEs	**7.30e+02**	1.81e+05	1.75e+04	1.30e+03
TP7	UFEs	**2.60e+02**	3.63e+03	4.04e+03	3.14e+02
	LFEs	**3.31e+03**	8.64e+05	2.68e+05	4.23e+04
TP8	UFEs	**2.36e+02**	2.94e+03	1.45e+03	2.66e+02
	LFEs	**1.42e+03**	3.19e+05	1.23e+04	1.88e+03
TP9	UFEs	**2.74e+02**	7.42e+02	6.60e+02	2.52e+02
	LFEs	**2.99e+04**	6.65e+05	9.66e+04	3.72e+04
TP10	UFEs	**8.29e+02**	5.47e+02	6.18e+02	7.77e+02
	LFEs	**1.92e+05**	8.99e+05	4.02e+05	3.01e+05

However, when the numbers of markets and distributors are increase, the profits of manufacturer and distributors gradually grow, and HBLSTA shows outstanding performance. In addition, from the total function evaluations (FEs), it can be seen that compared with BLEAQ2, HBLSTA can find a better solution with higher efficiency when solving the ML models.

Table 11 shows the numerical experiment results of the DL model. Similarly, HBLSTA can get more profits, no matter at the manufacturer or distributor level. In terms of algorithm efficiency, HBLSTA requires fewer FEs and is more efficient in solving DL model.

The results verify that HBLSTA can effectively solve the BLP models we proposed.

5.3. Model application

This section is dedicated to introducing the characteristics of the application of the BLP models (ML and DL) in actual cases.

The novel NBLP models are proposed by combining multiple coordination strategies and limiting the production budget at each interval. No relevant studies on the same issue have been found yet, so there is a lack of comparable data. In fact, in the model evaluation, we utilize real-world data from a lead-zinc supply chain in the non-ferrous metallurgy industry to test the performance of the algorithm. Therefore, the lead-zinc supply chain is also adopted as a case of our model application.

The parameters of manufacturer, distributors and markets are randomly selected from the uniform distribution of various parameters in the lead-zinc supply chain (see Table 9). The results are shown in Table 12.

In Table 12, the profits obtained by manufacturers, distributors and the overall supply chain in the ML and DL models are compared. It can be seen that manufacturer or distributors who play the leader role in the supply chain can often obtain higher profits, which is consistent with market rule. At the same time, although the profits obtained by manufacturers and distributors in the DL and ML models are quite different, the overall profits of the supply chain are very close. Moreover, it is worth noting that the profits of the distributors are much higher than those of the manufacturer in DL model. From the perspective of model construction, the main reason is that in the demand function (Eq. (2)), both advertising expenditure and selling price are determined by distributors. If distributors occupy the leading position in the supply chain, they can adjust their strategies to maximize their own interests. And coupled with the influence of the demand function, distributors have significant advantages in the distribution of benefits throughout the supply chain.

The above findings are consistent with cognition in real world. In the BLP models, the role of the leader is very important, and the leader with priority decision-making power can often obtain more profit share in the overall profit of the supply chain. Therefore, the status of each member in the supply chain must be reasonably allocated for different situations, and the member who have priority to maximize profits will play the role of leader.

Table 8
Efficiency comparison of the HBLSTA with the other three algorithms on the TP test suite. (best, worst and mean).

Pr.	Level	HBLSTA			NBLEA			BLEAQ			BLEAQ2		
		Best	Worst	Mean	Best	Worst	Mean	Best	Worst	Mean	Best	Worst	Mean
TP1	UFEs	**1.01e+02**	**1.65e+02**	**1.12e+02**	1.12e+03	1.33e+03	1.22e+03	7.28e+ 02	7.96e+ 02	7.77e+ 02	1.06e+02	1.86e+02	1.13e+02
	LFEs	**5.37e+02**	**5.62e+02**	**5.52e+02**	1.59e+05	1.62e+05	1.61e+05	1.49e+ 04	1.50e+ 04	1.50e+ 04	5.64e+02	5.89e+02	5.72e+02
TP2	UFEs	**1.09e+02**	**1.21e+02**	**1.12e+02**	3.22e+03	3.45e+03	3.36e+03	1.39e+03	1.44e+03	1.42e+03	2.35e+02	2.67e+02	2.42e+02
	LFEs	**5.89e+02**	**6.08e+02**	**5.93e+02**	2.43e+05	2.43e+05	2.43e+05	1.45e+04	1.45e+04	1.45e+04	1.76e+03	1.78e+03	1.77e+03
TP3	UFEs	**0.99e+02**	**1.12e+02**	**1.04e+02**	1.46e+03	1.49e+03	1.48e+03	3.56e+02	3.76e+02	3.63e+02	1.49e+02	1.66e+02	1.54e+02
	LFEs	**5.15e+02**	**5.25e+02**	**5.22e+02**	1.21e+05	1.21e+05	1.21e+05	4.46e+03	4.49e+03	4.48e+03	5.28e+02	5.35e+02	5.31e+02
TP4	UFEs	**2.95e+02**	**3.11e+02**	**3.02e+02**	1.75e+03	1.77e+03	1.76e+03	2.65e+02	2.80e+02	2.74e+02	2.91e+02	2.96e+02	2.93e+02
	LFEs	**2.72e+03**	**2.72e+03**	**2.72e+03**	2.73e+05	2.73e+05	2.73e+05	1.53e+04	1.53e+04	1.53e+04	3.31e+03	3.31e+03	3.31e+03
TP5	UFEs	**2.80e+02**	**2.86e+02**	**2.83e+02**	3.09e+03	3.09e+03	3.09e+03	1.29e+03	1.31e+03	1.30e+03	3.11e+02	3.30e+02	3.20e+02
	LFEs	**1.74e+03**	**1.74e+03**	**1.74e+03**	1.48e+05	1.48e+05	1.48e+05	1.57e+04	1.57e+04	1.57e+04	2.42e+03	2.43e+03	2.43e+03
TP6	UFEs	**1.33e+02**	**1.47e+02**	**1.40e+02**	3.25e+02	3.30e+02	3.28e+02	2.78e+02	2.99e+02	2.88e+02	1.77e+02	1.98e+02	1.83e+02
	LFEs	**7.22e+02**	**7.32e+02**	**7.30e+02**	1.81e+05	1.81e+05	1.81e+05	1.75e+04	1.75e+04	1.75e+04	1.30e+03	1.31e+03	1.30e+03
TP7	UFEs	**2.55e+02**	**2.80e+02**	**2.64e+02**	3.62e+03	3.63e+03	3.62e+03	4.02e+03	4.04e+03	4.03e+03	3.34e+02	3.01e+02	3.18e+02
	LFEs	**3.31e+03**	**3.31e+03**	**3.31e+03**	8.64e+05	8.64e+05	8.64e+05	2.68e+05	2.68e+05	2.68e+05	4.23e+04	4.23e+04	4.23e+04
TP8	UFEs	**2.12e+02**	**2.44e+02**	**2.36e+02**	2.92e+03	2.95e+03	2.94e+03	1.44e+03	1.45e+03	1.45e+03	2.61e+02	2.68e+02	2.66e+02
	LFEs	**1.42e+03**	**1.43e+03**	**1.42e+03**	3.19e+05	3.19e+05	3.19e+05	1.23e+04	1.23e+04	1.23e+04	1.86e+03	1.89e+03	1.88e+03
TP9	UFEs	**2.65e+02**	**2.82e+02**	**2.79e+02**	7.22e+02	7.52e+02	7.44e+02	6.52e+02	6.66e+02	6.61e+02	2.48e+02	2.53e+02	2.51e+02
	LFEs	**2.99e+04**	**2.99e+04**	**2.99e+04**	6.65e+05	6.65e+05	6.65e+05	9.66e+04	9.66e+04	9.66e+04	3.72e+04	3.72e+04	3.72e+04
TP10	UFEs	**8.13e+02**	**8.40e+02**	**8.30e+02**	5.32e+02	5.59e+02	5.46e+02	6.05e+02	6.31e+02	6.19e+02	7.62e+02	7.88e+02	7.76e+02
	LFEs	**1.92e+05**	**1.92e+05**	**1.92e+05**	8.99e+05	8.99e+05	8.99e+05	4.02e+05	4.02e+05	4.02e+05	3.01e+05	3.01e+05	3.01e+05

Table 9
Domains of problem parameters.

Manufacturer		Distributors		Markets	
Parameter	Domain	Parameter	Domain	Parameter	Domain
Pc	[1.2, 2]	Oc_j	[50, 250]	b_k	[30000, 230000]
Sc	[50, 250]	Dh_j	[0.05, 0.15]	α_k	[1.5, 2.7]
Mh	[0.05, 0.15]			β_k	[0.05, 0.85]
Tc_j	[0.1, 0.2]				

Table 10
Test results for ML model.

Test problem	Manufacturer's profit		Distributor's profit		FEs	
	HBLSTA	BLEAQ2	HBLSTA	BLEAQ2	HBLSTA	BLEAQ2
$J = 2, K = 3$	55,816	53,822	27,489	27,357	43,410	82,715
$J = 3, K = 5$	80,150	55,754	42,841	40,653	93,320	113,907
$J = 4, K = 7$	95,302	80,885	52,860	42,526	103,275	123,783
$J = 5, K = 10$	123,649	108,124	58,547	47,800	135,248	154,457
$J = 6, K = 15$	189,996	164,290	81,437	77,616	227,433	241,283

Table 11
Test results for DL model.

Test problem	Distributor's profit		Manufacturer's profit		FEs	
	HBLSTA	BLEAQ2	HBLSTA	BLEAQ2	HBLSTA	BLEAQ2
$J = 2, K = 3$	80,925	75,110	5,700	3,997	52,267	61,399
$J = 3, K = 5$	125,757	84,734	5,796	5,277	63,336	73,888
$J = 4, K = 7$	176,420	126,656	7,651	7,503	103,812	171,429
$J = 5, K = 10$	236,879	152,674	9,406	9,285	172,129	268,629
$J = 6, K = 15$	336,427	234,857	14,244	12,524	343,899	482,384

Table 12
Profit comparison in ML and DL models.

Test problem	Manufacturer's profit		Distributor's profit		Overall profit	
	ML	DL	ML	DL	ML	DL
$J = 2, K = 3$	63,457	4,880	31,507	89,435	94,964	94,315
$J = 3, K = 5$	83,315	7,662	57,222	134,970	140,537	142,632
$J = 4, K = 7$	91,556	12,671	64,390	142,914	155,946	155,585
$J = 5, K = 10$	126,337	24,387	75,811	177,993	202,148	202,380
$J = 6, K = 15$	163,822	30,524	83,886	217,037	247,708	247,561

In addition, different impacts on key resources including demand can also lead to differences in the profits earned by supply chain members.

6. Conclusion

This paper concentrate on the decentralized supply chain including one manufacturer and multiple distributors as the research object. The pricing, inventory management, advertising and transportation issues involved in supply chain coordination have been investigated and comprehensively considered. According to the characteristics of the decentralized supply chain problem, two BLP models have been established utilizing BLP technology, which are used to guide the formulation of supply chain coordination strategies. The market demands in the models are affected by the joint nonlinearity of selling price and advertising expenditure. Furthermore, the constraints of production budget in each production interval and advertising budget has also been concerned. In addition, in view of the difficulty and inefficiency of solving BLPPs, a bi-level optimization method (i.e., HBLSTA) based on hybrid STA and mapping approximation strategy has been developed in this paper. After verifying the effectiveness of the HBLSTA, it has been applied to solve the decentralized supply chain issues.

According to the numerical experiments of the benchmark instances, HBLSTA shows excellent solution accuracy and computational efficiency in solving NBLPPs. Moreover, HBLSTA can obtain an ideal equilibrium solution with higher efficiency when solving the BLP models of the decentralized supply chain. The results of the model evaluation show that in the decentralized supply chain models, the switching of the leader and follower roles will lead to differences in the distribution of benefits between manufacturer and distributors. Normally the supply chain member who play the role of leader will get more benefits. At the same time, the influence of individual members on demand will also change the profits they can obtain in the decentralized supply chain.

In the decentralized supply chain system studied in this paper, both upper and lower levels are single-objective optimization problems. However, considering the multi-objective problem in the actual supply chain, how to establish multi-objective BLP models and design the corresponding algorithms to guide supply chain coordination and decision-making could be among interesting future research directions. Furthermore, the pessimistic formulations of decentralized supply chain BLP models will also be an important research work in the future.

Declaration of competing interest

The authors declare that they have no known competing financial interests or personal relationships that could have appeared to influence the work reported in this paper.

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