

Upper bound analysis of slope stability with nonlinear failure criterion based on strength reduction technique

ZHAO Lian-heng(赵炼恒), LI Liang(李亮), YANG Feng(杨峰), LUO Qiang(罗强), LIU Xiang(刘项)

School of Civil and Architectural Engineering, Central South University, Changsha 410075, China

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Abstract: Based on the upper bound limit analysis theorem and the shear strength reduction technique, the equation for expressing critical limit-equilibrium state was employed to define the safety factor of a given slope and its corresponding critical failure mechanism by means of the kinematical approach of limit analysis theory. The nonlinear shear strength parameters were treated as variable parameters and a kinematically admissible failure mechanism was considered for calculation schemes. The **iterative optimization method** was adopted to obtain the safety factors. Case study and comparative analysis show that solutions presented here agree with available predictions when nonlinear criterion reduces to linear criterion, and the validity of present method could be illuminated. From the numerical results, it can also be seen that nonlinear parameter m , slope foot gradient β , height of slope H , slope top gradient α and soil bulk density γ have significant effects on the safety factor of the slope.

Key words: nonlinear failure criterion; strength reduction method; upper-bound theorem of limit analysis; slope stability analysis; factor of safety

1 Introduction

Soil slope stability analysis plays an increasingly indispensable role in the field of geotechnical as well as civil engineering, and it has aroused a lot of investigation. General methods for slope stability analysis are as follows: limit equilibrium method, limit analysis theorem, slip-line field method and numerical analysis method [1]. Limit analysis theory has been widely used because of its definite physical significance and strict solving range. However, the main evaluation indexes of limit analysis for slope stability are critical height (H_{cr}) and stability factor (N_s) at present, which differ from the universal evaluation index of safety factor (F_s), thus causing lots of inconvenience to the soil slope stability analysis [1–2]. The issue that combines limit analysis theory with the strength reduction technique to comprehensively analyze the stability of slope has been seldom considered [3–6].

Meanwhile, the linear Mohr–Coulomb (MC) failure criterion has been widely used in these efforts and techniques mentioned above. However, nearly all the experimental results show that the strength envelopes of almost all the geo-materials are characterized as nonlinear, and that linear failure criterion is a special case of failure criteria [7–9].

A number of researchers have employed nonlinear failure criterion to calculate critical height (H_{cr}) and stability factor (N_s) of slopes with limit analysis theory [7–12] and finite element method [13]; nonetheless, few studies [14] have obtained the safety factors by using the limit equilibrium method. However, limit equilibrium method is usually taken as a non-strict solution according to the randomness in block dividing and the assumption on inter-force between blocks [1].

For the reasons mentioned above, by using upper bound limit analysis and strength reduction technique, the main point of this work is to get the upper bound solution of safety factor (F_s) under the assumption of nonlinear failure criterion. The influences of nonlinear parameter m and other different parameters on slope safety factor (F_s) and latent slide surface were examined by using the iterative optimization method, and some charts of safety factor (F_s), which varied with nonlinear parameter m and other parameters, were presented for practical use in engineering.

2 Basic principle and assumptions

2.1 Strength reduction technique

Strength reduction technique was proposed by BISHOP in 1955 [15]. The shear strength parameters (c

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Corresponding author: ZHAO Lian-heng, PhD; Tel: +86–13755139425; E-mail: zlh8076@163.com

and φ) are divided by slope safety factor (F_s), which are analytically defined as Eq.(1), and make the slope reach a critical state.

$$\begin{cases} c_f = c/F_s \\ \varphi_f = \arctan(\tan \varphi/F_s) \end{cases} \quad (1)$$

where safety factor F_s serves as the reduction factor of shear strength parameters; c is the cohesive strength; φ is the internal friction angle; c_f denotes the reduced cohesive strength; and φ_f stands for the reduced internal friction angle.

2.2 Upper bound analysis based on strength reduction method for slope stability

According to strength reduction technique, by substituting the reduced shear strength parameters into the expression of virtual work principle, the limit analysis theory can be combined with reduction theorem following a certain stability criterion. For a slope, this upper bound analytic process can be described as follows: if a kinematically admissible failure mechanism is available, the safety factor is equal to 1.0 when the slope height arrives at critical height H_{cr} . Thus, under certain conditions of an actual slope height H_{actual} , the slope stays at stable state when the actual slope height H_{actual} is just right greater than or equal to critical height (H_{cr}) after deducing the strength parameters (c and φ). At this moment, $H_{actual}=H_{cr}$ can be regarded as the evaluation index of the slope stability, and the reduction factor of original strength parameters is precisely the safety factor (F_s) of slope stability [4–6].

2.3 Nonlinear yield criterion and energy dissipation

The experimental results show that strength envelopes of almost all geo-materials can be characterized as nonlinear in σ_n - τ stress space, while, a nonlinear M-C yield criterion can usually be expressed as [8]

$$\tau = c_0 \cdot (1 + \sigma_n/\sigma_t)^{1/m} \quad (2)$$

where σ_n and τ are normal and shear stresses on failure envelope (or surface), respectively; c_0 , σ_t and m are test parameters that can be measured by test. When nonlinear parameter $m=1$, Eq.(2) reduces to the well-known linear M-C yield criterion.

A limit load computed from a pyramidal failure surface, which always circumscribes the actual failure surface, will be an upper bound on the actual limit load [3]. Thus, the linear MC failure criterion represented by the tangential line will give an upper bound on the actual load for the material, whose failure is governed by the nonlinear failure criterion. By adopting this idea, a tangential line to the nonlinear yield criterion, is employed by YANG et al [10–12] to calculate the energy dissipation of geo-materials, thereby avoiding the

calculation difficulty under the nonlinear failure criterion. A more comprehensive description of this method can be found in Refs.[10–12].

Then, mobilized internal friction angle φ_t is introduced as an intermediate variable, in the form of $\tan \varphi_t = d\tau/d\sigma_n$, the tangential line to the curve at the location of tangency point can be expressed as

$$\tau = c_t + \tan \varphi_t \cdot \sigma_n \quad (3)$$

where c_t is the intercept of the tangential line on the τ -axis. c_t is determined by the following expression:

$$c_t = \frac{m-1}{m} \cdot c_0 \cdot \left(\frac{m \cdot \sigma_t \cdot \tan \varphi_t}{c_0} \right)^{\frac{1}{1-m}} + \sigma_t \cdot \tan \varphi_t \quad (4)$$

As for nonlinear failure criterion, the original strength indexes (c and φ) are to be altered into nonlinear shear strength indexes c_t and φ_t as tangential line method as follows:

$$\begin{cases} c_t = c_t/F_s \\ \varphi_t = \arctan(\tan \varphi_t/F_s) \end{cases} \quad (5)$$

2.4 Basic assumptions

In order to solve the stability problem of slopes, some assumptions have been made.

- (1) The slope is long enough. Therefore this problem can be regarded as a plane strain problem.
- (2) The filling is idealized as a perfectly plastic material, and follows the associated flow rule.
- (3) The rate of external work is due to soil weight, and the contribution to energy dissipation is provided along the failure surface.

3 Calculation for safety factor of slope

In this work, a rotational failure mechanism following a log-spiral slip surface is shown in Fig.1, where β' is the angle related line CC' to line AC' ; θ_0 is the angle related horizontal line to line OB ; and θ_h is the angle related horizontal line to line OC' . This mechanism, which is considered by CHEN [2], is geometrically defined by angles α , β , β' , θ_0 , θ_h and the mobilized internal friction angle φ_t .

Calculations of the rate of work dissipation and the work rate of the soil weight for rotational mechanism can be found in CHEN [2]. Equating the work rate of external forces to the internal energy dissipation rate, the objective function of safety factor F_s can be written as follows:

$$F_s = \frac{c_t}{\gamma H_{actual}} \cdot \frac{e^{2[(\theta_h - \theta_0) \cdot \tan \varphi_t]} - 1}{2 \cdot \tan \varphi_t \cdot (f_1 - f_2 - f_3 - f_4)} \cdot \left(\frac{H_{actual}}{r_0} \right) \quad (6)$$

where H_{actual} denotes the actual slope height; functions

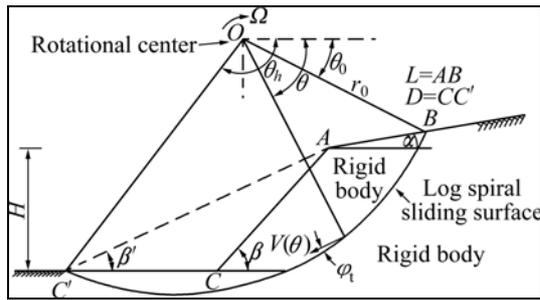


Fig.1 Rotational failure mechanism for slope stability analysis

for f_1 – f_4 depend on angles θ_h , θ_0 , α , β , β' , and ϕ_t , and can be found in CHEN[2]. According to the upper bound theorem, the solution of Eq.(6) falls into a classical optimization problem. The least upper bound for safety factor F_s can be found by solving the following set of equations:

$$\begin{cases} \frac{\partial F_s}{\partial \theta_h} = 0, & \frac{\partial F_s}{\partial \theta_0} = 0, & \frac{\partial F_s}{\partial \beta'} = 0, & \frac{\partial F_s}{\partial (\tan \phi_t)} = 0 \\ \theta_0 < \theta_h, & \beta' \leq \beta, & \tan \phi_t \geq 0, & H_{cr} = H_{actual} \end{cases} \quad (7)$$

In Eq.(7), the unknown quantities are θ_h , θ_0 , β' and ϕ_t ; in effect, safety factor F_s is an implicit function at the same time. So, iterative optimization calculation is adopted to obtain the least upper bound for safety factor F_s by reducing nonlinear shear strength indexes (c , and ϕ_t).

Once θ_h , θ_0 , β' , c_t and ϕ_t are found, the geometry of the critical failure surface is completely defined. L and D are the parameters used to draw the position of the potential sliding surface in Fig.1. There are

$$L = \left[\frac{\sin(\theta_h - \theta_0)}{\sin(\theta_h + \alpha) \cdot (H_{actual}/r_0)} - \frac{\sin(\theta_h + \beta')}{\sin(\theta_h + \alpha) \cdot \sin \beta'} \right] H_{actual} \quad (8)$$

and

$$D = \frac{\sin(\beta - \beta')}{\sin \beta \cdot \sin \beta'} \cdot H_{actual} \quad (9)$$

where L expresses the distance between the failure surface at crest and edge of the slope; D represents the distance between the failure surface at bottom of the slope and slope toe, and $D=0$ means the slipping surfaces passing through the slope toe.

4 Comparisons and analysis

4.1 Comparison with calculation on linear failure criterion

An embankment slope example based on linear failure criterion was cited to illustrate the validity of this method. The parameters in this example are as follows: slope height $H=6$ m, slope top gradient $\alpha=0^\circ$, slope foot

gradient $\beta=55^\circ$, soil bulk density $\gamma=18.6$ kN/m³, cohesion force $c=16.7$ kN/m² and internal friction angle $\phi=12^\circ$. Comparisons were made with different methods (simplified bishop method, Swedish slice method, Janbu method and finite difference method) on safety factor and latent slip surface, which are outlined in Fig.2.

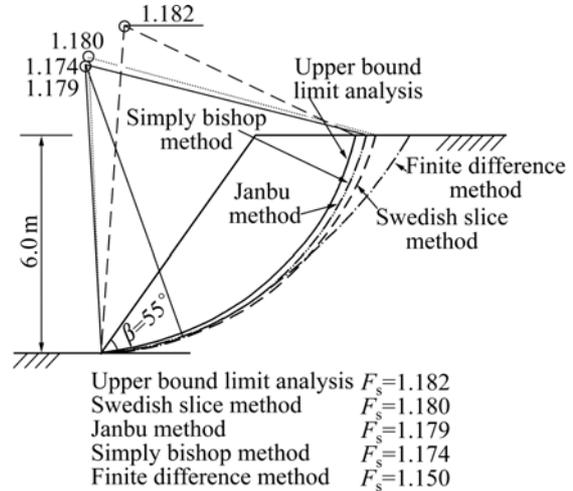


Fig.2 Comparison of different methods on critical sliding surfaces and safety factor based on linear failure criterion

As seen from Fig.2, slope safety factor obtained by present method is approximately the same as that by other methods, and absolute error is no more than 2.1%; the latent slip surfaces obtained by traditional limit-equilibrium methods are adjacent to each other, which confirm the validity of this method.

4.2 Comparison with calculation on nonlinear failure criterion

To show the validity of the present approach under the assumption of nonlinear failure criterion, the example, in which $\tau_0=90$ kN/m², $\sigma_t=247.3$ kN/m² were used by ZHANG and CHEN [7], was chosen like other authors including DRESCHER and CHRISTOPOULOS [8], COLLINS et al [9], YANG and YIN [10] and LI [13]. Table 1 shows a comparison of stability factor N_s obtained by ZHANG and CHEN [7] with the results of stability factor N_s , critical height H_{cr} and corresponding safety factor F_s computed by the present solution. For the cases analyzed, seven values of nonlinear parameter m are taken into account from 1.0 to 2.5 and four slopes are considered with slope foot gradient β varying from 45° to 90° . In addition, slope top gradient $\alpha=0^\circ$ and soil bulk density $\gamma=20.0$ kN/m³ are considered at the same time.

As shown in Table 1, for $\beta=90^\circ$, 75° , 60° and 45° with $m=1.0$ – 2.5 , the most absolute difference between the present computational stability factor N_s and that obtained by ZHANG and CHEN [7] is 0.72%, 1.51%, 2.94% and 7.43%, respectively. And all of the present stability factor N_s solutions are less than those obtained

by ZHANG and CHEN [7]. In terms of the upper bound limit analysis method, the smaller the stability factor N_s , the better the upper bound solution.

Theoretically speaking, given a slope with certain parameters, it is true that safety factor (F_s) is 1.0 when slopes reach critical height (H_{cr}) and stability factor (N_s) [2–6]. Also, when slopes reach critical height (H_{cr}) and stability factor (N_s), safety factors (F_s) for all of the cases mentioned above are illustrated in Table 1. From Table 1,

it is obvious that safety factor F_s is equal to 1.000 for all cases, which means that the proposed method is effective for evaluating slope stability by applying strength reduction technique under the condition of nonlinear failure criterion.

The corresponding effects of nonlinear parameter m on mobilized internal friction angle ϕ_t and the intercept of tangential line on τ -axis c_t are shown in Fig.3.

Fig.3 indicates that nonlinear parameter m has a

Table 1 Comparison of stability factors N_s obtained by ZHANG and CHEN [7] with results of stability factor N_s , critical height H_{cr} and corresponding safety factor F_s computed by present solution

m	$\beta=90^\circ$				$\beta=75^\circ$			
	Ref.[7]	This work			Ref.[7]	This work		
	N_s	N_s	H_{cr}/m	F_s	N_s	N_s	H_{cr}/m	F_s
1.0	5.51	5.505	24.771	1.000	7.48	7.476	33.643	1.000
1.2	5.13	5.118	23.192	1.000	6.77	6.707	30.557	1.000
1.4	4.89	4.872	22.150	1.000	6.33	6.249	28.629	1.000
1.6	4.73	4.703	21.413	1.000	6.04	5.950	27.316	1.000
1.8	4.60	4.581	20.867	1.000	5.82	5.739	26.367	1.000
2.0	4.52	4.488	20.446	1.000	5.66	5.583	25.650	1.000
2.5	4.35	4.332	19.723	1.000	5.40	5.328	24.445	1.000

m	$\beta=60^\circ$				$\beta=45^\circ$			
	Ref.[7]	This work			Ref.[7]	This work		
	N_s	N_s	H_{cr}/m	F_s	N_s	N_s	H_{cr}/m	F_s
1.0	10.39	10.390	46.754	1.000	16.18	16.159	72.715	1.000
1.2	8.95	8.784	40.428	1.000	12.55	12.045	56.723	1.000
1.4	8.13	7.923	36.790	1.000	10.82	10.242	48.916	1.000
1.6	7.61	7.393	34.437	1.000	9.70	9.241	44.300	1.000
1.8	7.24	7.036	32.794	1.000	9.10	8.608	41.253	1.000
2.0	6.97	6.779	31.582	1.000	8.78	8.173	39.093	1.000
2.5	6.54	6.373	29.602	1.000	7.95	7.515	35.708	1.000

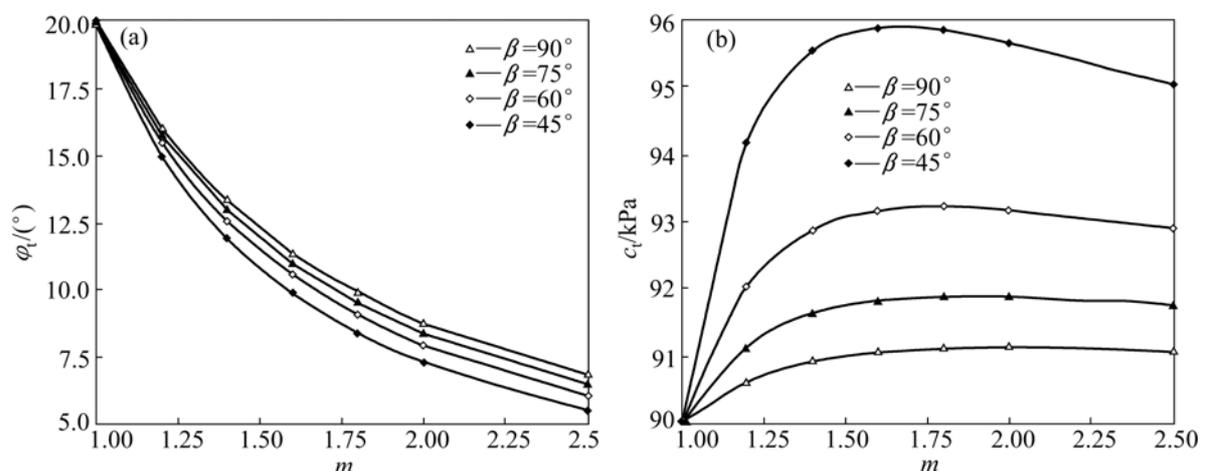


Fig.3 Effect of nonlinear parameter m on mobilized internal friction angle ϕ_t and intercept of tangential line on τ -axis c_t for different slopes with slope foot gradient β varying from 45° to 90° : (a) ϕ_t versus m ; (b) c_t versus m

notable influence on mobilized internal friction angle φ_t and the intercept of tangential line on τ -axis c_t for different slopes with slope foot gradient β varying from 45° to 90° . There is a fall in the mobilized internal friction angle φ_t as m goes up, and the intercept of tangential line on τ -axis c_t increases firstly followed by a fall with the increase in m , and the trend is more marked for a flat slope with $\beta=45^\circ$.

5 Parameters analysis

In order to investigate the effect of safety factor F_s of a slope when a nonlinear yield criterion is considered, and a series of cases are studied. The effects of nonlinear parameter m , slope foot gradient β , slope height H , slope top gradient α and soil bulk density γ on safety factor F_s and potential sliding surface of slopes are explored here.

5.1 Influence of nonlinear parameter m and slope foot gradient β on slope stability

Given a slope with $H=25$ m, $\alpha=5^\circ$, $\gamma=20$ kN/m³, $c_0=90$ kN/m², and $\sigma_t=247.3$ kN/m². When nonlinear parameter m ranges from 1.0 to 2.5, and slope foot gradient β varies from 30° to 90° , safety factors figured out by upper bound analysis are outlined in Fig.4. Under the same conditions, the critical failure surfaces coefficients (L/H , D/H) versus nonlinear parameter (m) for different slopes are shown in Fig.5 and the corresponding latent slip surfaces are shown in Fig.6.

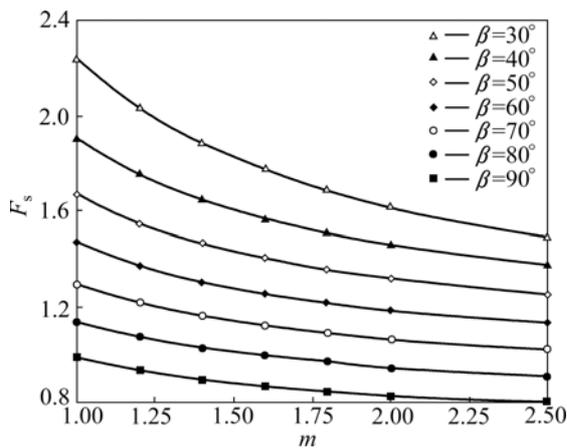


Fig.4 Effect of nonlinear parameter m on safety factor F_s for different slopes with β varying from 30° to 90°

It can be seen from Fig.4, there is a dramatic decline in safety factor with the rise of m . More importantly, slope foot gradient β affects F_s evidently as well. With regard to a flat slope with $\beta=30^\circ$, m varying from 1.0 to 2.5, the absolute decrease in the safety factors F_s (from 2.238 8 to 1.482 6) amounts to 51.0%; and for vertical cut ($\beta=90^\circ$), m ranging from 1.0 to 2.5, the absolute decrease in safety factor F_s is 22.9% (from 0.986 8 to

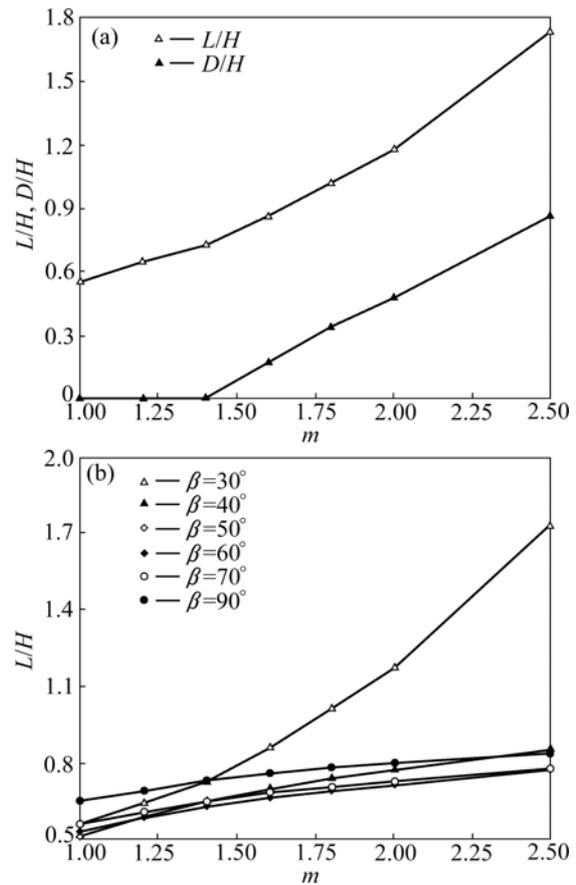


Fig.5 Critical failure surfaces coefficients (L/H and D/H) versus nonlinear parameter (m) for different slopes: (a) $\beta=30^\circ$; (b) $\beta=30^\circ-90^\circ$

to 0.802 8).

According to Figs.5 and 6, when other parameters are fixed, the latent slide surface moves inwardly to slope as m ascends. It seems that, whilst the sliding surface becomes bigger, the failure mass becomes much larger than that in linear case ($m=1.0$). Meanwhile, there is a significant drop in safety factor with increasing m . Regarding a flat slope with $\beta=30^\circ$, with m changing from 1.0 to 2.5, the value of L/H shoots up to 212.4% (from 0.556 2 to 1.737 8); when it comes to vertical cut case ($\beta=90^\circ$), there is a absolute increase of 29.2% in L/H , from 0.647 3 ($m=1.0$) to 0.836 4 ($m=2.5$).

Similarly, as shown in Figs.5 and 6, slope foot gradient β influences latent slide surface, that is, the smaller the slope foot gradient β , the larger the influence degree. For a certain slope with β larger than or equal to 30° , all the outcomes of these cases, in which slipping surfaces pass through the slope toe and D/H equals 0 are described in Figs.6(b)–(d). Failure surfaces nearly have no intersection into foundation as for linear failure criterion, in addition, the failure surfaces slightly penetrate into the foundation ground as m rises. But when it comes to a flat slope with $\beta=30^\circ$, with m ranging from 1.0 to 2.5, the slipping surfaces pass through the

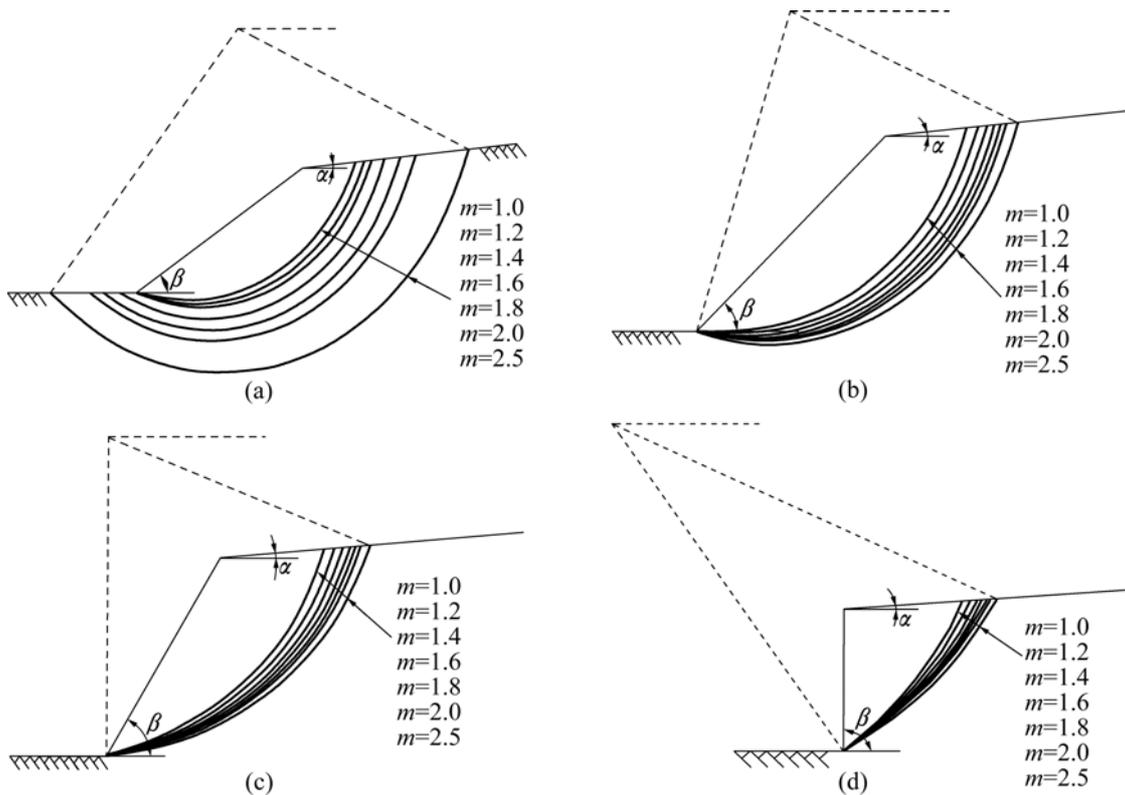


Fig.6 Comparison of potential sliding surface based on nonlinear failure criterion for different β values: (a) $\beta=30^\circ$; (b) $\beta=40^\circ$; (c) $\beta=60^\circ$; (d) $\beta=90^\circ$

slope toe firstly and then deeply penetrate into foundation ground.

5.2 Influence of nonlinear parameter m and height of slope H on slope stability

Given a slope with $\beta=60^\circ$, $\alpha=10^\circ$, $\gamma=20 \text{ kN/m}^3$, $c_0=90 \text{ kN/m}^2$, $\sigma_t=247.3 \text{ kN/m}^2$. When nonlinear parameter m has a variety of values ($m=1.0-2.5$) and slope height H varies from 10 to 30 m, safety factor F_s figured out by upper bound analysis is outlined in Fig.7. Under the same conditions, the corresponding critical failure surfaces coefficient (L/H) versus the nonlinear parameter (m) for different slopes are shown in Fig.8.

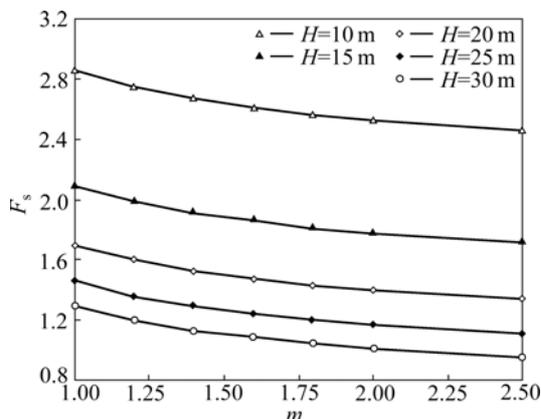


Fig.7 Effect of nonlinear parameter m on safety factor F_s for different slopes with H varying from 10 to 30 m

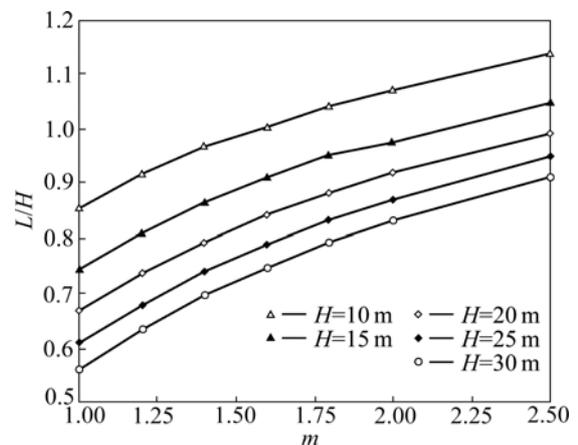


Fig.8 Effect of nonlinear parameter m on critical failure factor L/H for different slopes with H varying from 10 to 30 m

As shown in Fig.7, safety factor F_s decreases with the increase of m and H , moreover, the influence of H becomes more obvious with the increase of H . Regarding a single slope with $H=20 \text{ m}$, $m=1.0$ to $m=2.5$, safety factor F_s descends by 27.0% (from 1.695 8 to 1.335 7); as for a single slope with the nonlinear parameter $m=2.0$, H varying from 10 to 30 m, the absolute decrease in safety factor F_s is as high as 150.99% (from 2.526 5 to 1.006 6).

According to Fig.8, when other parameters remain unchanged under nonlinear failure criterion, latent slide surface moves towards the inner-slope with the increase

of m . At the same time, the sliding surface becomes bigger, and the failure mass is far larger than that in the linear case ($m=1.0$). Slope height H influences the latent slide surface to some extent, to be more specific, the smaller the slope height H , the larger the L/H . In terms of a single slope with $H=20$ m, the value of L/H rises by 48.9%, from 0.666 6 ($m=1.0$) to 0.992 7($m=2.5$); whereas, for a single slope with nonlinear parameter $m=1.8$, there is a fall of 31.3% in L/H , from 1.039 1 at $H=10$ m to 0.791 5 at $H=30$ m.

5.3 Influence of nonlinear parameter m and slope top gradient α on slope stability

Assuming a slope with $\beta=75^\circ$, $H=20$ m, $\gamma=20$ kN/m³, $c_0=90$ kN/m², $\sigma_t=247.3$ kN/m², the influence of nonlinear parameter m and slope top gradient α on slope stability is studied in this section. For the cases analyzed, seven values of nonlinear parameter m are taken into account from 1.0 to 2.5 and five slopes are considered with slope top gradient α varying from 0° to 20° , and safety factor F_s is outlined in Fig.9. Under the same conditions, the curves of critical failure surfaces coefficient (L/H) versus nonlinear parameter (m) for different slopes are illustrated in Fig.10 in detail and the corresponding latent slip surfaces are shown in Fig.11.

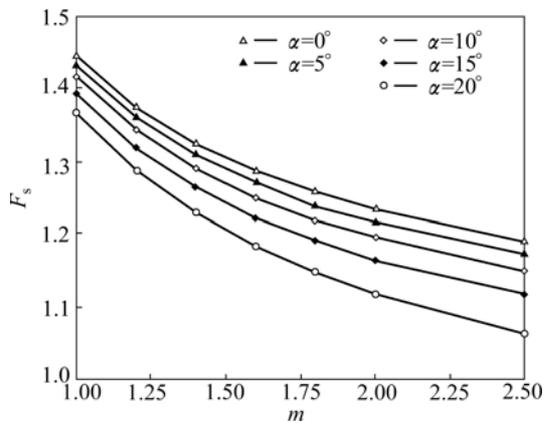


Fig.9 Effect of nonlinear parameter m on safety factor F_s for different slopes with α varying from 0° to 20°

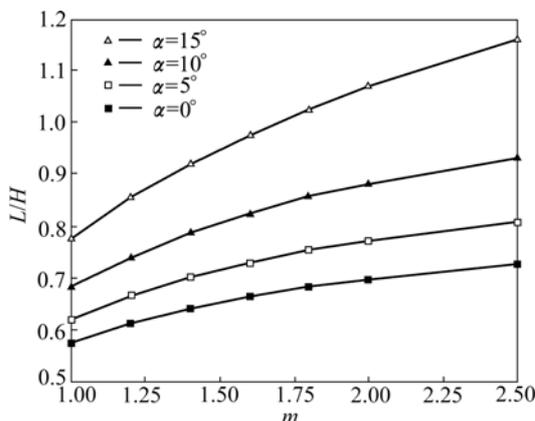


Fig.10 Effect of nonlinear parameter m on failure surface coefficient L/H with α varying from 0° to 15°

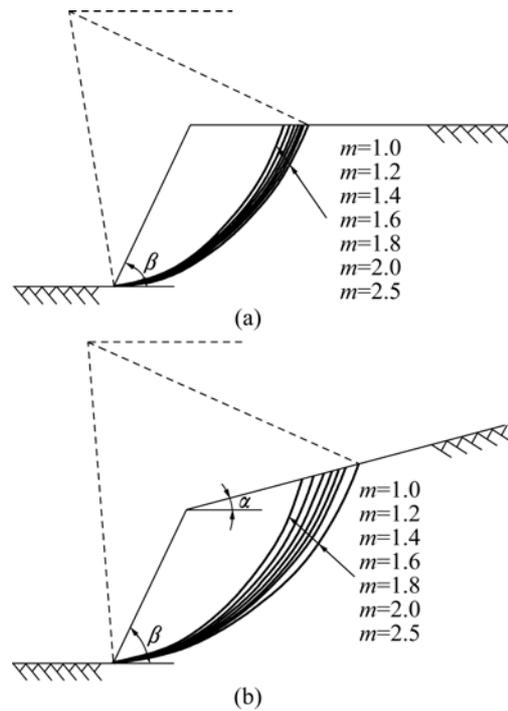


Fig.11 Comparison of potential sliding surface based on nonlinear failure criterion: (a) $\alpha=0^\circ$; (b) $\alpha=15^\circ$

It is seen from Fig.9 that there is an inverse correlation between safety factor F_s and m , as well as that between F_s and α , but the change of F_s influenced by m is greater than that by α . When it comes to a single slope with $\alpha=15^\circ$, m varying from 1.0 to 2.5, the absolute decrease in safety factor F_s is 24.7% (from 1.393 9 to 1.117 9); as for a single slope with the nonlinear parameter $m=2.0$, α ranging from 0° to 20° , safety factor F_s shows a decline of 10.3%, from 1.233 7 ($\alpha=0^\circ$) to 1.118 7($\alpha=20^\circ$).

According to Figs.10 and 11, when other parameters are invariable under nonlinear failure criterion, the latent slide surface moves inward to the slope with the increase of m and α , in the meantime, sliding surface becomes bigger, failure mass is modestly larger than the linear case ($m=1.0$), and safety factor declines gradually with the increase of m simultaneously.

Slope top gradient α has a vital effect on latent slide surface as well, for a single slope with nonlinear parameter $m=2.0$, α ranging from 0° to 15° , the absolute increase in L/H reaches 53.3% (from 0.697 8 to 1.069 4).

5.4 Influence of nonlinear parameter m and soil bulk density γ on slope stability

Given a slope with $\beta=75^\circ$, $\alpha=10^\circ$, $H=20$ m, $c_0=90$ kN/m², $\sigma_t=247.3$ kN/m². Both nonlinear parameter m and soil bulk density γ affect slope stability. For the cases analyzed, seven values of nonlinear parameter m are taken into account from 1.0 to 2.5 and slopes are

considered with soil bulk density γ varying from 16 to 26 kN/m³, and safety factor F_s is figured out in Fig.12. Under the same conditions, the curves of critical failure surface coefficient (L/H) versus nonlinear parameter (m) for different slopes are shown in Fig.13 and the corresponding latent slip surfaces are shown in Fig.14.

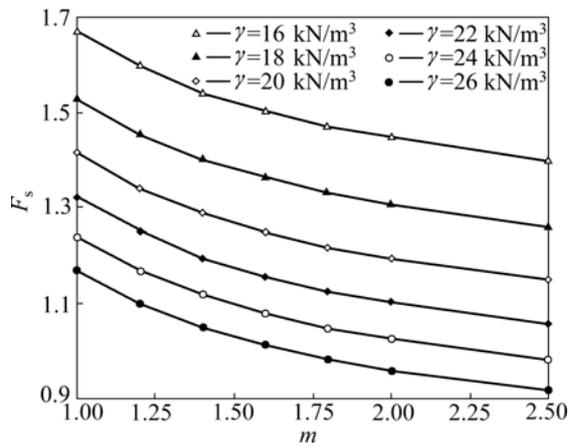


Fig.12 Effects of nonlinear parameter m on safety factor F_s for different slopes with γ varying from 16 to 26 kN/m³

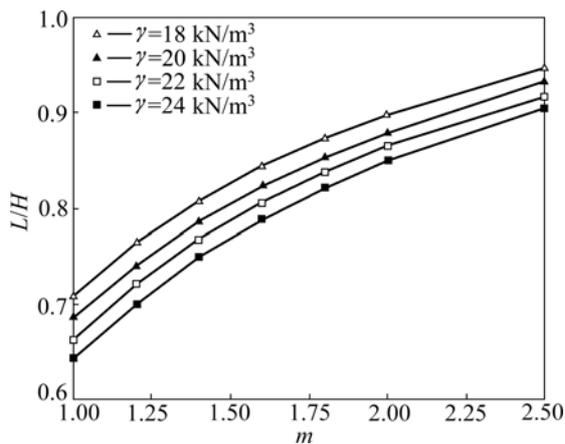


Fig.13 Critical failure surface coefficient L/H versus nonlinear parameter m for different slopes with γ varying from 18 to 24 kN/m³

It is shown from Fig.12 that safety factor F_s declines with increasing m and rising γ , additionally, there is an increasing rate in the rise of F_s influenced by γ . For a single slope with $\gamma=20$ kN/m³, m varying from 1.0 to 2.5, the safety factor F_s falls by 23.0% (from 1.415 0 to 1.150 2); and for a single slope with nonlinear parameter $m=2.0$, and γ from 16 to 26 kN/m³, there is an absolute decrease of 50.1% (from 1.445 6 to 0.960 0) in safety factor F_s with the γ ranging from 16 to 26 kN/m³.

According to Figs.13 and 14, when other parameters are fixed under nonlinear failure criterion, the latent slide surface moves inward to the slope with the increase of m , sliding surface becomes bigger, failure mass is a little bit larger than that in linear case ($m=1.0$), and safety factor declines gradually with the increase of m at the same time.

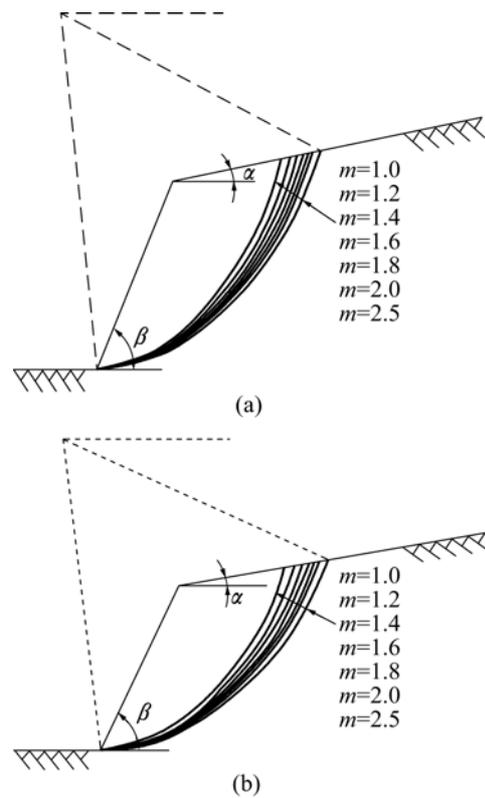


Fig.14 Comparison of potential sliding surface based on nonlinear failure criterion: (a) $\gamma=20$ kN/m³; (b) $\gamma=24$ kN/m³

Soil bulk density γ has a certain effect on latent slide surface as well, for a single slope with nonlinear parameter $m=2.0$, L/H experiences an decrease of 5.7% (from 0.898 5 to 0.850 0), with γ varying from 18 to 24 kN/m³.

6 Conclusions

(1) Case study and comparative analysis show that solutions presented here agree with available predictions when nonlinear criterion reduces to linear criterion, and the validity of present method can be illuminated.

(2) The effects of nonlinear parameter m , slope foot gradient β , height of slope H , slope top gradient α and soil bulk density γ are investigated by using log-spiral rotational fracture surface in upper bound method with nonlinear yield criterion. It is found that all these parameters have a significant influence on slope stability. Under the same conditions, safety factor F_s of slopes decreases non-linearly as the stability condition of a slope deteriorates (when an increase in values of α , β , H and γ occurs).

(3) The nonlinear strength parameter value of soil has a vital influence on evaluation of slope stability, the safety factor F_s of slopes decreases non-linearly and a deeper failure surface with a larger sliding wedge is observed with the rise in value of m ; consequently, it is

crucial to introduce the assumption of nonlinear strength curve into the geo-material analysis.

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