Joined Influences of Nonlinearity and Dilation on the Ultimate Pullout Capacity of Horizontal Shallow Plate Anchors by Energy Dissipation Method

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Abstract: The ultimate pullout capacity (UPC) and the shape modification factors of horizontal plate anchors were calculated by using upper-bound limit analysis, in which the assumptions of both a nonlinear failure criterion and the nonassociated flow rule were made upon the soil mass above the anchor plate. Three types of anchor plates, including strip anchors, circle anchors, and rectangle anchors, and the corresponding failure mechanisms are taken into consideration. The anchor breakout factors were obtained according to the principle of virtual power, which was realized numerically by the nonlinear sequential quadratic programming algorithm. The shape modification factors for different kinds of anchors were given through a multiple nonlinear regression method. Numerical experiments demonstrate the validity of the solutions by reducing the solutions (nonlinear criterion and nonassociated flow rule) into their special cases (linear criterion and associated flow rule), which matches well with existing work. The dilation and nonlinearity of soil mass should be considered because it plays a remarkable role in the UPC of anchor plates. **DOI: 10.1061/(ASCE)GM.1943-5622.0000075.** *© 2011 American Society of Civil Engineers.*

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Introduction

Soil plate anchors are commonly used as foundation systems for structures that require uplift or lateral resistance, such as transmission towers, sheet pile walls, offshore oil wells, and gas facilities. As the use of plate anchors expands, a greater understanding of their behavior, including the ultimate pullout capacity (UPC), is required. Various research methods have been used to study the bearing behavior of plate anchors. The main types of research methods are (1) full-scale and/or scaled-model tests (Meyerhof and Adams 1968; Murray and Geddes 1987; Murray et al. 1989; Ilamparuthi et al. 2002; Dickin and Laman 2007); (2) numerical simulation techniques (Rowe and Davis 1982; Merifield et al. 2006; Dickin and Laman 2007); (3) the limit equilibrium method (Meyerhof and Adams 1968; Ghaly and Hanna 1994); and (4) lower-bound and upper-bound theorem of limit analysis (Murray and Geddes 1987; Merifield et al. 2006; Shi and Zhao 2011; Zhao et al. 2009a, b).

The preceding researchers have carried out great efforts to analyze the UPC on the basis of the linear failure criterion and the associated flow rule. However, in the case of dense granular materials, a key factor in constitutive behavior is described as the characteristic of dilatancy angle, which is different from friction angle (Davis 1968; Zienkiewicz et al. 1975; Chen and Liu 1990; Drescher and Detournay 1993). According to the plasticity theory of geomaterials, dilatancy means the geomaterials follow the nonassociated flow rule, which influences the UPC. Therefore, a number of researchers have employed the nonassociated flow rule to calculate the UPC of anchor plates (Rowe and Davis 1982; Murray and Geddes 1989; Ilamparuthi et al. 2002; Merifield et al. 2006; Dickin and Laman 2007).

Experiments have also shown the strength envelopes of geomaterials have the nature of nonlinearity (Agar et al. 1987; Drescher and Christopoulos 1988). The friction angle in most soils, particularly in granular soils, decreases with the increase of confining pressures; in other words, the Mohr envelope is curved. In addition, the adoption of the associated flow rule, on which the limit analysis method is established, results in an overprediction of soil dilation (Yang and Huang 2009).

In regards to the effect of nonlinearity of failure criterion on the slope stability and earth pressure problems, Drescher and Christopoulos (1988) and Yang and Yin (2006) proposed a generalized tangential technique to calculate the slope stability and earth pressure of a retaining wall with a nonlinear failure criterion on the basis of the upper-bound theorem. On the basis of limit analysis theory, Yang and Huang (2009) applied the slice method to evaluate the slope stability problem by taking the joint influences of nonlinearity and dilation into consideration. Shi and Zhao (2011) analyzed the uplift features of vertically loaded strip plate anchors based on the same principle.

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Because of the preceding reasons, this paper extends Yang and Huang's (2009) and Shi and Zhao's (2011) research to the calculation of UPC and the shape factors for different kinds of plate anchors by adopting a tangential technique, which considers a nonlinear yield criterion and the nonassociated flow rule. Charts of anchor breakout factors and shape modification factors for different kinds of anchors are presented.

Nonlinear Failure Criterion

Experimental results show the strength envelopes of almost all geomaterials have the nature of nonlinearity in $\sigma_n - \tau$ stress space. In general, a nonlinear yield criterion can be expressed as

$$\tau = c_0 \cdot (1 + \sigma_n/\sigma_t)^{1/m} \tag{1}$$

where c_0, σ_t and $m(\geq 1)$ = test parameters and can be obtained by triaxial test.

The sketch map of a nonlinear yield criterion is shown in Fig. 1, where σ_n and τ = normal and shear stresses on the failure envelope, respectively. When the nonlinear parameter $m = 1$, Eq. (1) reduces to the well-known linear Mohr-Coulomb yield criterion. A more detailed introduction of this nonlinear yield criterion can be found in Drescher and Christopoulos (1988), Yang and Yin (2006), and Yang and Huang (2009).

A limit load computed from a convex failure surface, which always circumscribes the actual failure surface, will be the upper bound on the actual limit load. Thus, the linear Mohr-Coulomb failure criterion represented by a tangential line will give an upper bound on the actual load for a material whose failure is governed by a nonlinear failure criterion. By using this idea, a tangential line to the nonlinear yield criterion is employed by Drescher and Christopoulos (1988), Yang and Yin (2006), and Yang and Huang (2009) to calculate the energy dissipation of geomaterials and avoid the calculation difficulty under a nonlinear failure criterion. The tangential line to the curve at the location of tangency point G, as shown in Fig. 1, can be expressed as $\tau = c_t + \sigma_n \tan\varphi_t$, where φ_t = mobilized internal friction angle as an intermediate variable and can be introduced as $\tan\varphi_t = d\tau/d\sigma_n$. The term c_t = intercept of the tangential line on the τ-axis and can be expressed as

$$c_t = (m-1)/m \cdot c_0 \cdot [(m \cdot \sigma_t \cdot \tan\varphi_t)/c_0]^{1/(1-m)} + \sigma_t \cdot \tan\varphi_t \tag{2}$$

A more comprehensive description of this method can be found in Drescher and Christopoulos (1988), Yang and Yin (2006), and Yang and Huang (2009) and is not repeated here.

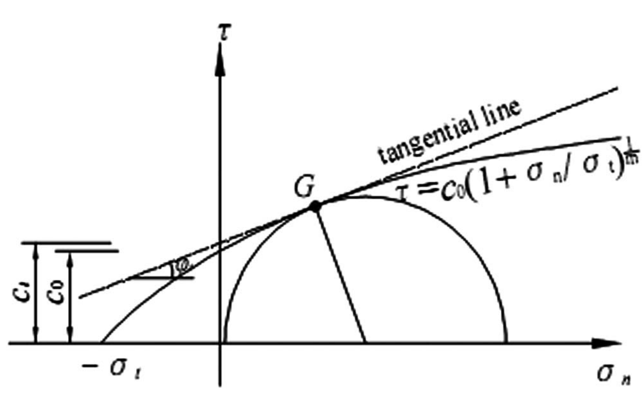

Fig. 1. Curve for a nonlinear failure criterion

Upper-Bound Analysis

The upper-bound theorem in limit analysis is expressed as follows. The loads for the stability of the system can be estimated by a kinematically admissible failure mechanism when the power of the loads applied to the system is larger than the power that can be dissipated within the system during its movement.

$$\int_S F_i v_i^* dS + \int_A \gamma_i v_i^* dA = \int_A \sigma_{ij}\dot{\varepsilon}_{ij}^* dA + \int_{S_D} (\tau - \sigma_n \tan\varphi)\Delta v_i^* ds \tag{3}$$

where F_i = limit load that induced the failure of the system; and $F_i \leq$ actual load F at this moment. To simplify the calculation process, the plastic region can be divided into several sliding blocks that are detached by sliding surfaces. The deformations of the sliding blocks are in accordance with one another in the plastic region when plastic failure occurs. Therefore, the sliding blocks can be regarded as rigid blocks, and the energy dissipation does not occur among the blocks. A more comprehensive description of this method can be found in Chen (1975) and Chen and Liu (1990).

The soil mass above the anchor plate is assumed to follow not only the perfect plastic assumptions, but also the nonassociative assumptions, and it satisfies the coaxial flow rule as well, which means that the axes of the increments of principle plastic strains coincide the axes of principle stresses in the process of plastic deformation. The soil dilation is characterized by the dilation angle $\eta = \psi/\varphi_t$ (Drescher and Detournay 1993; Wang et al. 2001; Simoni and Houlsby 2006; Yang and Huang 2009).

Nonassociated Flow Rule

The plasticity theory indicates material follows a nonassociated flow rule if the dilatancy angle is not identical to the friction angle. According to the nonassociated flow rule, the velocity at velocity discontinuities inclines a dilatancy angle with respect to the velocity discontinuity line. For a nonlinear failure criterion material, the dilatancy angle varies from zero to the internal friction angle φ_t. Correspondingly, the dilative coefficient η, which relates the dilatancy angle to the soil friction angle, is defined as $\eta = \psi/\varphi_t$, where φ_t = friction angle and ψ = dilatancy angle. Theoretically, the magnitude of dilative coefficient is $0 \leq \eta \leq 1$. The case $\eta = 1$ indicates the material follows the associated flow rule. For a nonlinear failure criterion material following the coaxial nonassociated flow rule, the following equations represent the dilatancy of nonlinear materials:

$$c^* = c_t \frac{\cos\psi \cos\varphi_t}{1 - \sin\varphi_t \sin\psi} \tag{4}$$

$$\tan\varphi^* = \tan\varphi_t \frac{\cos\psi \cos\varphi_t}{1 - \sin\varphi_t \sin\psi} \tag{5}$$

where c^* and φ^* = modified cohesion and friction angles for the upper-bound analysis when the material follows the coaxial nonassociated flow rule and a nonlinear failure criterion. Eqs. (4) and (5) are valid for the situation in which geomaterials are assumed to follow the tangential line failure criterion. A more detailed description of this method can be found in Yang et al. (2007) and Yang and Huang (2009).

Compatible Velocity and Energy Dissipation

Several studies supposed types of anchors as shallow anchors when $H/B \leq 8$, and the integral damage occurred through the soil mass above the anchor (Murray 1987; Ilamparuthi 2002; Dickin 2007). The ratio H/B is called the critical embedment ratio, where

H = depth of embedment, and B = width or diameter of the anchor plate. The reasonable velocity failure mechanism under the effect of vertical force for a horizontal shallow strip anchor is shown in Fig. 2, which was constructed by Murray and Geddes (1987).

Because the velocities for the translational failure mechanism are known, the work rate done by the external load and the internal energy dissipation rates can be calculated. Because the soil mass is regarded as perfectly rigid with no general plastic deformation, the internal energy is dissipated only along the velocity discontinuity surfaces af, bf, ce, and de, and the four parts are calculated in sum as G_z. The external work rate is done by external loads such as soil self-weight and UPC, which are expressed as W_{soil} and $P_u V_0$, respectively.

By using the same process, the failure mechanisms for the circle anchor plates and rectangular anchor plates could be constructed, as shown in Fig. 3.

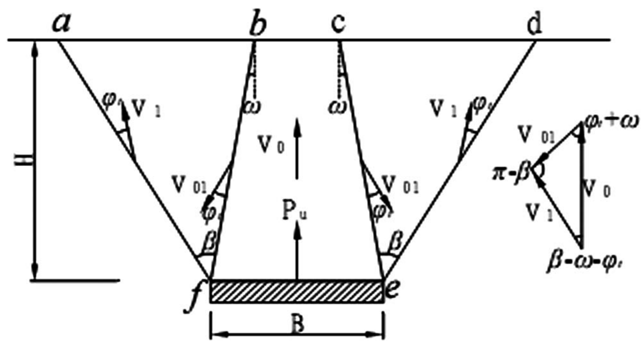

Fig. 2. Failure mechanism and velocity hodograph of strip anchors (data from Murray and Geddes 1987)

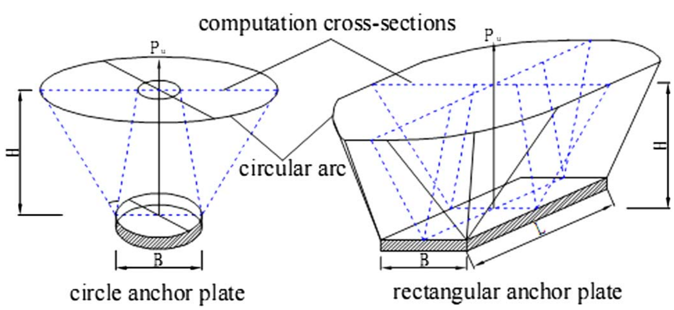

Fig. 3. Failure mechanism of 3D horizontal anchor plates

By equating the work done by external forces to the dissipation of energy, the upper-bound solution of UPC of a strip anchor plate in soil could be expressed as

$$P_u = (G_z - W_{\mathrm{soil}})/V_0 \tag{6}$$

where P_u = UPC of the anchor plate; V_0 = velocity of the anchor plate under the effect of the UPC; and φ_t, ω, β = angle parameters.

According to the upper-bound theorem of limit analysis, the ultimate pullout capacity P_u can be obtained by minimization of these coefficients with respect to the mechanism parameters and the location of the tangential point. The dimensionless anchor breakout factor N_{Pu} was defined as $N_{Pu} = P_u/\gamma AH$, in which γ = soil weight above the anchor plates, and A = area of the anchor plates. Thus, the magnitude of the anchor breakout factor N_{Pu} not only depends on the parameters of nonlinear failure criterion including m, c_0 and σ_t, but it also depends on the dilative parameter η and angle parameters ω and β.

Numerical Results

Here, the calculations of UPC are based on the assumption of a nonlinear failure criterion and the nonassociated flow rule. Eq. (6) provides an upper-bound solution of UPC for different kinds of anchor plates. The numerical results for this problem are obtained by using a generalized tangential technique. An optimization procedure with a sequential quadratic programming algorithm is employed to get a least upper bound of the anchor breakout factor N_{Pu} for different kinds of anchor plates with respect to m, c_0, σ_t, ω, β, φ_t, and η.

In the light of the preceding case study, comparisons were made of both linear/nonlinear failure criterion pairs and associated/nonassociated flow rule pairs to discuss the effects of nonlinearity (failure criterion) and dilation (nonassociated flow rule) on the anchor breakout factor N_{Pu}. Finally, shape modification factors for different kinds of anchors are presented by using a multiple nonlinear regression method.

Comparison

A comparison of the present method and other solutions considering the influences of a linear failure criterion and the associated flow rule is shown in Table 1.

On the basis of the numerical results, the present anchor breakout factor N_{Pu} agrees well with those obtained from previous studies when the nonlinear criterion reduces to a linear criterion ($m = 1$) and the nonassociated flow rule reduces to the associated

Table 1. Comparison of N_{Pu} Calculated Values in Sand by Various Methods

			H/B							
	Plate type	Method	1	2	3	4	5	6	7	8
N_{Pu}	Strip anchor ($\varphi = 45°$)	Meyerhof	1.95	2.90	3.85	4.80	5.75	6.70	7.65	8.60
		Murray	2.00	3.00	4.00	5.00	6.00	7.00	8.00	9.00
		Present paper	2.00	3.00	4.00	5.00	6.00	7.00	8.00	9.00
	Circular anchor ($\varphi = 40°$)	Merifield	4.38	9.05	16.30	23.75	36.25	48.75	64.95	81.90
		Merifield	3.75	8.13	14.38	21.72	32.2	44.4	57.55	70.60
		Murray	3.61	7.75	13.05	19.00	27.05	36.65	46.65	57.85
		Ghaly	3.50	10.05	15.01	26.00	37.00	47.55	61.75	75.56
		Present paper	3.62	8.12	14.48	22.73	32.86	44.87	58.75	74.51
	Square anchor ($\varphi = 40°$)	Merifield	3.35	7.00	12.05	18.45	26.45	35.35	48.50	60.25
		Murray	3.42	7.31	12.67	19.51	27.82	37.61	48.88	61.61
		Present paper	3.00	6.59	11.65	18.19	26.20	35.68	46.64	59.07

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flow rule ($\eta = 1$). As a result, the comparison shows the proposed method is an effective method for evaluating the UPC of anchor plates.

Effect of Nonlinear Parameters

The nonlinear failure criterion and the nonassociated flow rule influence the anchor breakout factor (N_{Pu}). Fig. 4 shows N_{Pu} for five different kinds of anchor plates corresponding to $B = 1.0$ m, $c_0 = 20$ kPa, and $\sigma_t = 32$ kPa. The nonlinear coefficient m varies from 1.0–3.0, there are two different dilative parameters ($\eta = 0.5$ and $\eta = 1$), and two different embedment ratios ($H/B = 1$ and $H/B = 4$) have been considered. It has also been assumed in the calculations for all kinds of anchors, $B = 1.0$ m, and for the rectangular anchor plates, $L/B = 4$ or $L/B = 8$.

Figs. 4 and 5 show the nonlinear coefficient m has significant influence on the anchor breakout factor (N_{Pu}), and the anchor breakout factor presents a nonlinear decrease with the increase of the nonlinear coefficient m when the dilative parameter η is constant. Therefore, the linearity simplification of nonlinear geomaterials has a disadvantageous influence on evaluating the real pullout characteristics of anchor plates.

Effect of Dilation Parameters

To investigate how the anchor breakout factor (N_{Pu}) is influenced by the nonassociated flow rule with a nonlinear failure criterion, Figs. 6 and 7 illustrate the effects of the dilative parameter η ($\eta = 0.0$–1.0) for two nonlinear coefficients ($m = 1.5$ and $m = 2.0$) and two different embedment ratios ($H/B = 1$ and $H/B = 4$) on the anchor breakout factor (N_{Pu}) at $c_0 = 20$ kPa, $\sigma_t = 32$ kPa, and $B = 1.0$ m.

Figs. 6 and 7 show the anchor breakout factor (N_{Pu}) with the nonassociated flow rule is less than that with the associated flow rule, and the anchor breakout factor increases with the increase of dilative parameter η.

Several further conclusions from Table 1 and Figs. 4–7 are outlined as follows: First, for rectangular anchor plates with increasing length-width ratios (L/B), the value of N_{Pu} gradually tends closer to the N_{Pu} of strip anchor plates. Therefore, when L/B is greater than a certain value, a rectangular anchor plate can be analyzed as a strip anchor plate. Second, the anchor types exert significant influence on the N_{Pu} when other parameters are constant. The N_{Pu} for strip anchors is the least and the value of N_{Pu} for circular anchors is the greatest for the same conditions.

Dimensionless Shape Factor Analysis for the Associated Flow Rule

The effect of anchor shape on N_{Pu} can be expressed as a dimensionless factor

$$F_{\text{shape}} = N_{Pu\_3D}/N_{Pu\_\text{strip}} \tag{7}$$

where N_{Pu_3D} = breakout factor for different kinds of 3D anchor plates; N_{Pu_strip} = breakout factor for strip anchor plates with the

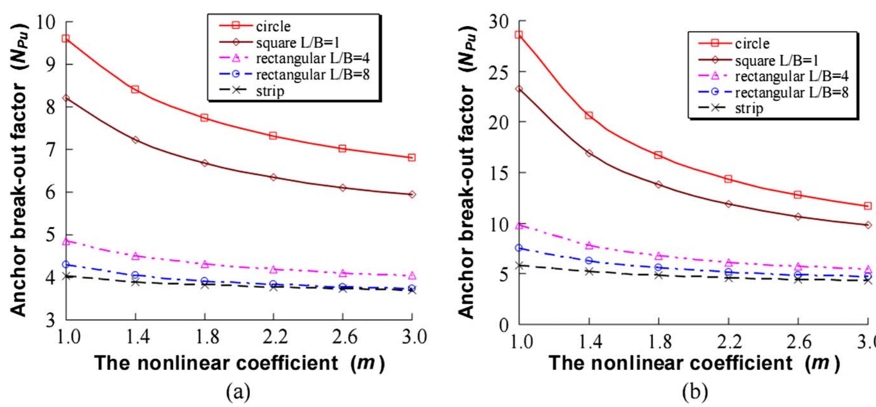

Fig. 4. Effects of nonlinear parameter m on anchor breakout factor N_{Pu} of different kinds of anchor plates at $\eta = 0.5$ for the embedment ratio (H/B): (a) $H/B = 1$; (b) $H/B = 4$

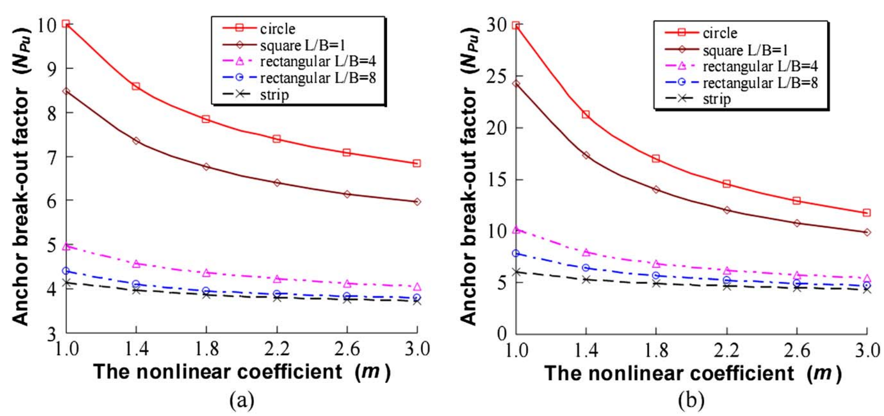

Fig. 5. Effects of nonlinear parameter m on anchor breakout factor N_{Pu} of different kinds of anchor plates at $\eta = 1.0$ for the embedment ratio (H/B): (a) $H/B = 1$; (b) $H/B = 4$

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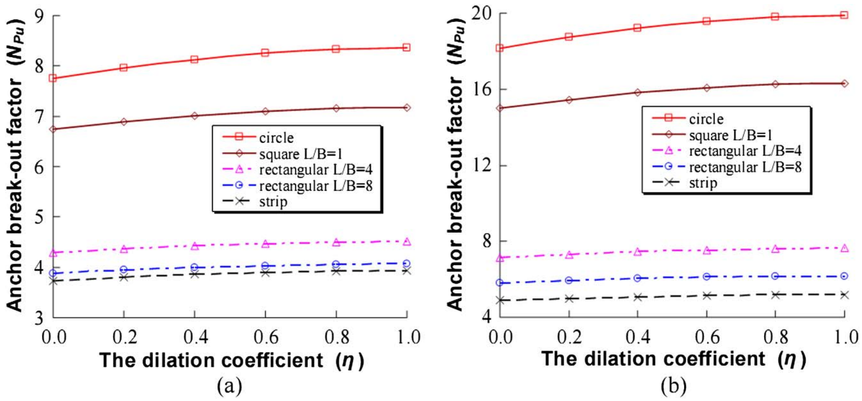

Fig. 6. Effects of dilative parameter η on anchor breakout factor N_{Pu} of different kinds of anchor plates at $m = 1.5$ for the embedment ratio (H/B): (a) $H/B = 1$; (b) $H/B = 4$

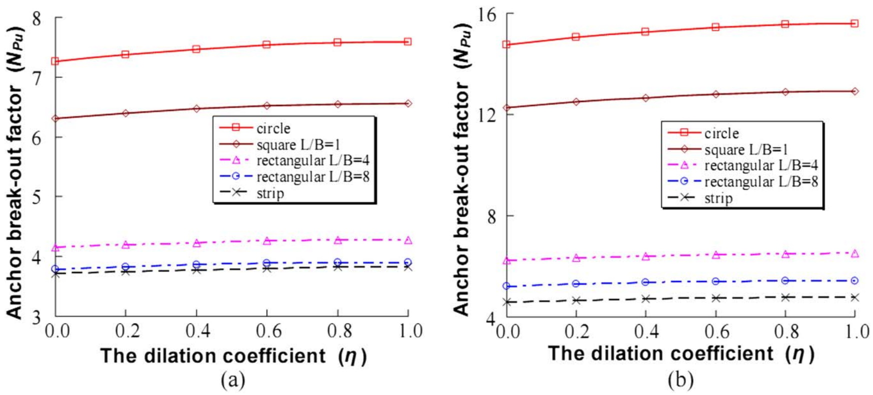

Fig. 7. Effects of dilative parameter η on anchor breakout factor N_{Pu} of different kinds of anchor plates at $m = 2.0$ for the embedment ratio (H/B): (a) $H/B = 1$; (b) $H/B = 4$

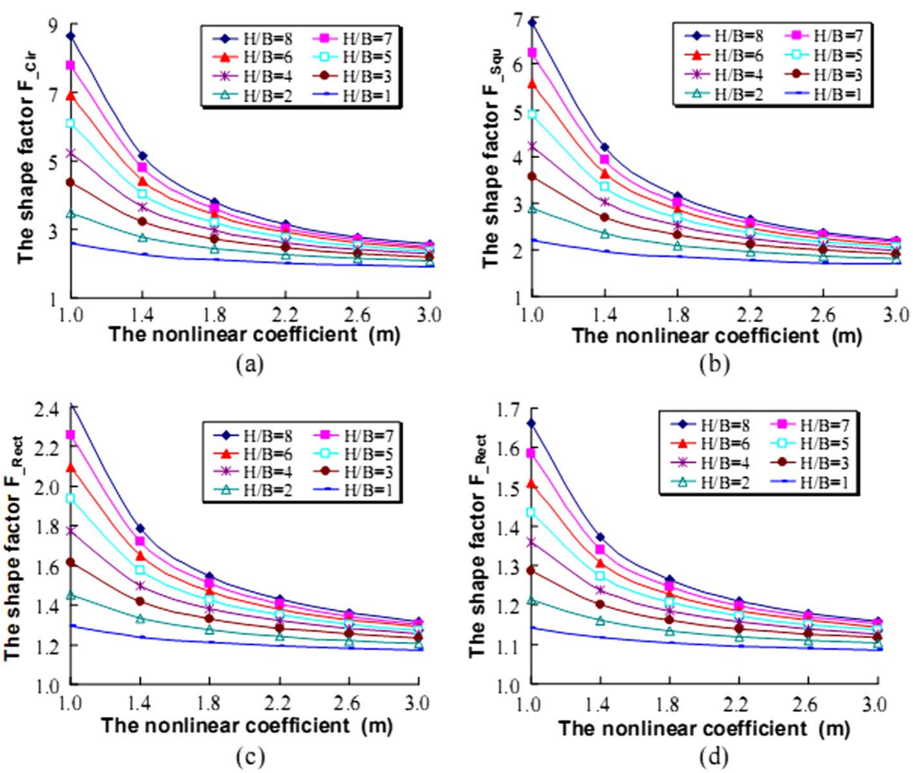

Fig. 8. Shape factor F_{shape} of different kinds of anchor plates for different embedment ratios

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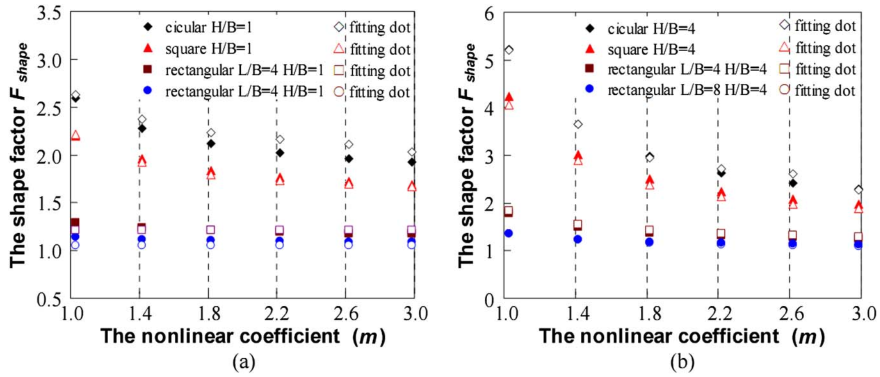

Fig. 9. Shape factor F_{shape} of different kinds of anchor plates for the embedment ratio (H/B): (a) $H/B = 1$; (b) $H/B = 4$

same calculation parameters (B, m, c_0, σ_t, and H/B) and the associated flow rule ($\eta = 1.0$). Several F_{strip} values for different kinds of 3D anchor plates were obtained with a sequential quadratic programming algorithm and are shown in Figs. 8(a)–8(d).

Through a multiple nonlinear regression method, we expect the shape factor to be expressed as a function of L/B, H/B, and m in the following approximate forms:

$$F_{\text{Cir}} = (3.68 - 4.44\,m + 1.89m^2 - 0.27m^3)\left(\frac{H}{B}\right) + (0.39 + 2.26m - 1.03m^2 + 0.15m^3) \qquad (8)$$

$$F_{\text{Rect}} = \frac{0.65 \cdot (H/B)}{(L/B) \cdot m^{2.0/[(L/B)^{0.05}]}} + \frac{1.6}{(L/B)^{0.2}} \qquad (9)$$

where the diameter of the circular anchor is equal to the strip anchor width; and if $F_{\text{Rect}} < 1.0$, let $F_{\text{Rect}} = 1.0$, which means the rectangular anchor can be analyzed as a strip anchor. Fig. 9 shows F_{shape} for different kinds of anchor plates corresponding to the embedment ratios $H/B = 1.0$ and $H/B = 4.0$.

It can be observed from Eqs. (8) and (9) and Fig. 9 that L/B, H/B, and m all have certain influences on the shape factor (F_{shape}) under the conditions of the associated flow rule ($\eta = 1$). In Eqs. (8) and (9), the shape factor (F_{shape}) is inversely linear proportional to the nonlinear coefficient m and is linear proportional to the embedment ratio (H/B), but the shape factor is inversely nonlinear proportional to the length-width ratio (L/B). The approximate functions of the shape factor compare reasonably well with the upper-bound calculation results as shown in Fig. 9.

Conclusions

On the basis of the preceding study, the following conclusions are drawn:

1. The UPC and the shape modification factors of different kinds of anchor plates have been studied with the upper-bound limit analysis theory by incorporating a nonlinear failure criterion and the nonassociated flow rule. Case studies and comparative analysis show the solutions presented here agree with available predictions when the nonlinear criterion reduces to the linear criterion ($m = 1$), and the nonassociated flow rule reduces to the associated flow rule ($\eta = 1$). Thus, the present method is validated.
2. Anchor type, embedment ratio (H/B), and material characteristics have significant influence on the pullout behavior of anchor plates. Under the condition of the associated flow rule ($\eta = 1$), the shape factor can be expressed as a function of the shape parameter L/B or D/B, H/B, and m by using a multiple nonlinear regression method. The anchor breakout factor of different kinds of anchor plates presents a nonlinear decreasing relationship with the increase of the nonlinear coefficient m when the other parameters are constant.

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