Supplemental material for "Integrating Variable Reduction Strategy with Evolutionary Algorithms for Solving Nonlinear Equations Systems"

The specific expressions, decision space, the actual known roots and the selected reduction schemes for F01-E46 are as follows:

(1) E1:

$$\begin{cases} x_1^2 - x_2^2 = 0 & (1) \\ 1 - |x_1 - x_2| = 0 & (2) \end{cases} \tag{1}$$

Where $x_i \in [-3,3]$, $i = 1,2$. It has two roots: (-0.5,0.5) and (0.5,-0.5).

The equation (1) is the eliminated equation. x_1 is the reduced variable. We can get the following variable reduction scheme:

$$x_1 = \pm x_2 \tag{2}$$

(2) E2:

$$\begin{cases} x_1^2 - x_2 - 2 = 0 & (1) \\ x_1 + \sin(\frac{\pi x_2}{2}) = 0 & (2) \end{cases} \tag{3}$$

Where $x_i \in [-3,3]$, $i = 1,2$. It has three roots: (1,-1), (0,-2) and (0.7075,-1.5).

The equation (2) is the eliminated equation. x_1 is the reduced variable. We can get the following variable reduction scheme:

$$x_1 = -\sin(\frac{\pi x_2}{2}) \tag{4}$$

(3) E3:

$$\begin{cases} x_1 x_2 + (x_1 - 2x_3)(x_2 - 2x_3) - 165 = 0 & (1) \\ \frac{x_1 x_2^3}{12} - \frac{(x_1 - 2x_3)(x_2 - 2x_3)^3}{12} - 9369 = 0 & (2) \\ \frac{2(x_2 - x_3)^2 (x_1 - x_3)^2 x_3}{x_1 + x_2 - 2x_3} - 6835 = 0 & (3) \end{cases} \tag{5}$$

Where $x_i \in [0,50]$, $i = 1,2,3$. It has two roots: (43.155566,10.128950,12.944048) and (7.602995,24.541982,11.576716).

The equation (2) is the eliminated equation. x_1 is the reduced variable. We can get the following variable reduction scheme:

$$x_1 = \frac{(x_3(x_2 - 2x_3)^3)/6 - 9369}{(x_2 - 2x_3)^3 - x_2^3/12} \tag{6}$$

(4) E4:

$$\begin{cases} x_1 + x_2 - 3 = 0 & (1) \\ x_1^2 + x_2^2 - 9 = 0 & (2) \end{cases} \tag{7}$$

Where $x_i \in [-3,3]$, $i = 1,2$. It has two roots: (0,3) and (3,0).

The equation (1) is the eliminated equation. x_1 is the reduced variable. We can get the following variable reduction scheme:

$$x_1 = 3 - x_2 \tag{8}$$

(5) E5:

$$\begin{cases} x_1 - \sin(2x_1 + 3x_2) - \cos(3x_1 - 5x_2) = 0 & (1) \\ x_2 - \sin(x_1 - 2x_2) + \cos(x_1 + 3x_2) = 0 & (2) \end{cases} \tag{9}$$

Where $x_i \in [-3,3]$, $i = 1,2$. It has three roots: (-0.173346,-0.256091), (0.838835,0.537119) and (0.792693,0.138017).

(6) E6:

$$\begin{cases} e^{x_1^2} - 8x_1 \sin(x_2) = 0 & (1) \\ x_1 + x_2 - 1 = 0 & (2) \\ (x_3 - 1)^3 = 0 & (3) \end{cases} \tag{10}$$

Where $x_i \in [0,1]$, $i = 1,2,3$. It has two roots: (0.673584,0.326416,1) and (0,1,1).

The equation (2) and (3) are the eliminated equations. x_2 and x_3 are the reduced variables. We can get the following variable reduction scheme:

$$\begin{aligned} x_1 &= 1 - x_2 \\ x_3 &= 1 \end{aligned} \tag{11}$$

(7) E7:

$$\begin{cases} x_1^3 - 3x_1 x_2^2 - 1 = 0 & (1) \\ 3x_1^2 x_2 - x_2^3 + 1 = 0 & (2) \end{cases} \tag{12}$$

Where $x_i \in [-1,2]$, $i = 1,2$. It has three roots: (-0.290515,1.084215), (1.084215,-0.290515) and (-0.793701,-0.793701).

The equation (1) is the eliminated equation. x_2 is the reduced variable. We can get the following variable reduction scheme:

$$x_2 = \pm\sqrt{\frac{x_1^3 - 1}{3x_1}} \tag{13}$$

(8) E8:

$$\begin{cases} x_1^2 + x_2^2 - 1 = 0 & (1) \\ x_1 - x_2 = 0 & (2) \end{cases} \tag{14}$$

Where $x_i \in [-1,1]$, $i = 1,2$. It has two roots: (-0.707107,-0.707107) and (0.707107，0.707107) .

The equation (2) is the eliminated equation. x_1 is the reduced variable. We can get the following variable reduction scheme:

$$x_2 = x_1 \tag{15}$$

(9) E9:

$$\begin{cases} \sum_{i=1}^{D} x_i^2 - 1 = 0 & (1) \\ | x_1 - x_2 | + \sum_{i=3}^{D} x_i^2 = 0 & (2) \end{cases} \tag{16}$$

Where $x_i \in [-1,1]$, $i = 1,2,\ldots,20$. It has two roots: $(-0.707107,-0.707107,0,\cdots,0)$ and $(0.707107,0.707107,0,\cdots,0)$.

The equation (1) is the eliminated equation. x_2 is the reduced variable. We can get the following variable reduction scheme:

$$x_2 = \pm\sqrt{1 - (x_1^2 + \sum_{i=3}^{D} x_i^2)} \tag{17}$$

(10) E10:

$$\begin{cases} x_1 - \sin(5\pi x_2) = 0 & (1) \\ x_1 - x_2 = 0 & (2) \end{cases} \tag{18}$$

Where $x_i \in [-1,1]$, $i = 1,2$. It has 11 roots: (-0.924840, -0.924840), (-0.866760, -0.866760), (-0.562010, -0.562010), (-0.428168, -0.428168), (-0.187960, -0.187960), (0.000000, 0.000000), (0.924840, 0.924840), (0.866760, 0.866760), (0.562010, 0.562010), (0.428168, 0.428168) and (0.187960, 0.187960).

The equation (2) is the eliminated equation. x_2 is the reduced variable. We can get the following variable reduction scheme:

$$x_2 = x_1 \tag{19}$$

(11) E11:

$$\begin{cases} x_1 - \cos(4\pi x_2) = 0 & (1) \\ x_1^2 + x_2^2 = 1 & (2) \end{cases} \tag{20}$$

Where $x_i \in [-1,1]$, $i = 1,2$. It has 15 roots: (0.416408, -0.909178), (-0.561364, -0.827569), (-0.724322, -0.689463), (0.837812, -0.545959), (0.886984, -0.461799), (-0.962322, -0.271914), (-0.972855, -0.231415), (1.000000, 0.000000), (0.416408, 0.909178), (-0.561364, 0.827569), (-0.724322, 0.689463), (0.837812, 0.545959), (0.886984, 0.461799), (-0.962322, 0.271914) and (-0.972855, 0.231415).

The equation (1) is the eliminated equation. x_1 is the reduced variable. We can get the following variable reduction scheme:

$$x_1 = \cos(4\pi x_2) \tag{21}$$

(12) E12:

$$\begin{cases} \cos(2x_1) - \cos(2x_2) - 0.4 = 0 & (1) \\ 2(x_2 - x_1) + \sin(2x_2) - \sin(2x_1) - 1.2 = 0 & (2) \end{cases} \tag{22}$$

Where $x_i \in [-10,10]$, $i = 1,2$. It has 13 roots: (-9.268258, -8.931402), (-8.744542, -7.164787), (-6.126665, -5.789809), (-5.602950, -4.023195), (-2.985073, -2.648216), (-2.461357, -0.881602), (-0.156520, 0.493376), (0.680236, 2.259991), (3.298113, 3.634969), (3.821828, 5.401583), (6.439705, 6.776562), (6.963421, 8.543176) and (9.581298,9.918154).

(13) E13- Interval arithmetic benchmark:

$$\begin{cases} x_1 - 0.25428722 - 0.18324757 x_4 x_3 x_9 = 0 & (1) \\ x_2 - 0.37842197 - 0.16275449 x_1 x_{10} x_6 = 0 & (2) \\ x_3 - 0.27162577 - 0.16955071 x_1 x_2 x_{10} = 0 & (3) \\ x_4 - 0.19807914 - 0.15585316 x_7 x_1 x_6 = 0 & (4) \\ x_5 - 0.44166728 - 0.19950920 x_7 x_6 x_3 = 0 & (5) \\ x_6 - 0.14654113 - 0.18922793 x_8 x_5 x_{10} = 0 & (6) \\ x_7 - 0.42937161 - 0.21180486 x_2 x_5 x_8 = 0 & (7) \\ x_8 - 0.07056438 - 0.17081208 x_1 x_7 x_6 = 0 & (8) \\ x_9 - 0.34504906 - 0.19615740 x_{10} x_6 x_8 = 0 & (9) \\ x_{10} - 0.42651102 - 0.21466544 x_4 x_8 x_1 = 0 & (10) \end{cases} \tag{23}$$

Where $x_i \in [-2,2]$, $i = 1,2,\cdots,10$. It has one root: (0.257833, 0.381097, 0.278745, 0.200669, 0.445251, 0.149184, 0.432010, 0.073403, 0.345967, 0.427326).

The equation (1), (6), (7) and (10) are the eliminated equations. x_1, x_6 , x_7 and x_{10} are the reduced variables. We can get the following variable reduction scheme:

$$\begin{aligned} x_1 &= 0.18324757 x_3 x_9 x_6 + 0.25428722 \\ x_6 &= 0.00744364 x_3 x_5 x_9 x_4^2 x_8^2 + 0.01032932 x_5 x_4 x_8^2 + 0.08070780 x_5 x_8 + 0.14654113 \\ x_7 &= 0.21180486 x_2 x_5 x_8 + 0.42937161 \\ x_{10} &= 0.03933692 x_3 x_8 x_9 x_4^2 + 0.05458668 x_8 x_4 + 0.42651102 \end{aligned} \tag{24}$$

(14) E14:

$$\begin{cases} 100(x_1-0.25)=0 & (1) \\ 100(x_1\sin(4\pi x_2^2)+0.75x_1-0.25)=0 & (2) \end{cases} \tag{25}$$

Where $x_i \in [-1,1]$, $i=1,2$. It has 8 roots: (0.250000, -0.854337), (0.250000, -0.721185), (0.250000, -0.479471), (0.250000, -0.141801), (0.250000, 0.141801), (0.250000, 0.479471), (0.250000, 0.721185) and (0.250000, 0.854337).

The equation (1) is the eliminated equation. x_1 is the reduced variable. We can get the following variable reduction scheme:

$$x_1 = 0.25 \tag{26}$$

(15) E15:

$$\begin{cases} 3-x_1x_3^2=0 & (1) \\ x_3\sin(\dfrac{\pi}{x_2})-x_3-x_4=0 & (2) \\ -x_2x_3e^{(1-x_1x_3)}+0.2707=0 & (3) \\ 2x_1^2x_3-x_2^4x_3-x_2=0 & (4) \end{cases} \tag{27}$$

Where $x_i \in [0,5]$, $i=1,2,\cdots,4$. It has one root: (3, 2, 1, 0).

The equation (1), (2) and (3) are the eliminated equations. x_2 x_3 and x_4 are the reduced variables. We can get the following variable reduction scheme:

$$\begin{aligned} x_2 &= \frac{0.2707}{\sqrt{\dfrac{3}{x_1}}e^{(1-x_1\sqrt{\frac{3}{x_1}})}} \\ x_3 &= \sqrt{\frac{3}{x_1}} \\ x_4 &= \sqrt{\frac{3}{x_1}}\sin\left(\frac{\pi\sqrt{\dfrac{3}{x_1}}e^{(1-x_1\sqrt{\frac{3}{x_1}})}}{0.2707}-\sqrt{\frac{3}{x_1}}\right) \end{aligned} \tag{28}$$

(16) E16:

$$\begin{cases} (1-\mathrm{R})\left(\dfrac{\mathrm{H}}{10(1+\beta_1)}-x_1\right)e^{\left(\frac{10x_1}{1+\frac{10x_1}{\gamma}}\right)}-x_1=0 & (1) \\ (1-\mathrm{R})\left(\dfrac{\mathrm{H}}{10}-\beta_1x_1-(1+\beta_2)x_2\right)e^{\left(\frac{10x_2}{1+\frac{10x_2}{\gamma}}\right)}+x_1-(1+\beta_2)x_2=0 & (2) \end{cases} \tag{29}$$

Where $x_i \in [0,1]$, $i=1,2$, $\mathrm{R}=0.96$, $\mathrm{H}=11$, $\gamma=1000$, $\beta_1=\beta_2=2$. It has 7 roots: (0.042100, 0.061813), (0.042100, 0.268723), (0.266600, 0.178430), (0.266600, 0.327267), (0.266600, 0.461111), (0.042318, 0.686779) and (0.719074, 0.244164).

The equation (2) is the eliminated equation. x_1 is the reduced variable. We can get the following variable reduction scheme:

$$x_1 = -(3x_2+\frac{1}{25}\mathrm{e}^{\frac{10x_2}{1+\frac{x_2}{100}}}(3x_2-\frac{11}{5}))/(\frac{2}{25}\mathrm{e}^{\frac{10x_2}{1+\frac{x_2}{100}}}-1) \tag{30}$$

(17) E17:

$$\begin{cases} 2x_1+x_2+x_3+x_4+x_5-6.0=0 & (1) \\ x_1+2x_2+x_3+x_4+x_5-6.0=0 & (2) \\ x_1+x_2+2x_3+x_4+x_5-6.0=0 & (3) \\ x_1+x_2+x_3+2x_4+x_5-6.0=0 & (4) \\ x_1x_2x_3x_4x_5-1.0=0 & (5) \end{cases} \tag{31}$$

Where $x_i \in [-10,10]$, $i=1,2,\cdots,5$. It has three roots: (1,1,1,1,1), (0.916355,0.916355,0.916355,0.916355,1.418227) and (-0.579043, -0.579043, -0.579043, -0.579043,8.895215).

The equation (1), (2), (3) and (4) are the eliminated equations. x_1, x_2, x_3 and x_4 are the reduced variables. We can get the following variable reduction scheme:

$$x_1 = x_2 = x_3 = x_4 = \frac{6-x_5}{5} \tag{32}$$

(18) E18:

$$\begin{cases} x_1+x_2^4x_4x_6/2+0.75=0 & (1) \\ x_2+0.405e^{1+x_1x_2}-1.405=0 & (2) \\ x_3-x_4x_6/2+1.5=0 & (3) \\ x_4-0.605e^{1-x_3^2}=0.395=0 & (4) \\ x_5-x_2x_6/2+1.5=0 & (5) \\ x_6-x_1x_5=0 & (6) \end{cases} \tag{33}$$

Where $x_i \in [-1,1]$, $i=1,2,\cdots,6$. It has one root: $(-1,1,1,1,-1,1)$.

The equation (1) (3), (5) and (6) are the eliminated equations. x_1, x_4, x_5 and x_6 are the reduced variables. We can get the following variable reduction scheme:

$$\begin{aligned} x_1 &= -0.5x_1^4x_2 - 0.75x_1^4 - 0.75 \\ x_4 &= \frac{1}{3}\frac{(2x_2+3)(3.0x_1+2x_1^5x_2+3x_1^5+8)}{(2x_1^4x_2+3x_1^4+3)} \\ x_5 &= \frac{-12}{(3x_1+2x_1^5x_2+3x_1^5+8)} \\ x_6 &= \frac{3(2x_1^4x_2+3x_1^4+3)}{(3x_1+2x_1^5x_2+3x_1^5+8)} \end{aligned} \quad (34)$$

(19) E19:

$$\begin{cases} \sin(x_1^3) - 3x_1x_2^2 - 1 = 0 & (1) \\ \cos(3x_1^2x_2) - |x_2^3| + 1 = 0 & (2) \end{cases} \quad (35)$$

Where $x_i \in [-2,2],\ i = 1,2$. It has 10 roots: (-1.810885, -0.349092), (-1.810885, 0.349092), (-1.502221, -0.409077), (-1.502221, 0.409077), (-1.791302, 0.301926), (-1.791302, -0.301926), (-0.947268, 0.785020), (-0.947268, -0.785020), (-0.213057, 1.256845) and (-0.213057, -1.256845).

The equation (1) is the eliminated equation. x_2 is the reduced variable. We can get the following variable reduction scheme:

$$x_2 = \pm\sqrt{\frac{\sin(x_1^3) - 1}{3x_1}} \quad (36)$$

(20) E20:

$$\begin{cases} 4x_1^3 + 4x_1x_2 + 2x_2^2 - 42x_1 - 14 = 0 & (1) \\ 4x_2^3 + 2x_1^2 + 4x_1x_2 - 26x_2 - 22 = 0 & (2) \end{cases} \quad (37)$$

Where $x_i \in [-5,5],\ i = 1,2$. It has 9 roots: (-0.127961, -1.953715), (-0.270845, 2.884255), (0.086678, 0.073852), (3.385154, -1.848127), (3.584428, -1.848127), (3.000000, 2.000000), (-3.779310, -3.283186), (-3.073026, -0.081353) and (-2.805118,3.131313).

The equation (1) is the eliminated equation. x_2 is the reduced variable. We can get the following variable reduction scheme:

$$x_2 = -x_1 \pm \sqrt{-2x_1^3 + x_1^2 + 21x_1 + 7} \quad (38)$$

(21) E21:

$$\begin{cases} -\sin(x_1)\cos(x_2) - 2\cos(x_1)\sin(x_2) = 0 & (1) \\ -\cos(x_1)\sin(x_2) - 2\sin(x_1)\cos(x_2) = 0 & (2) \end{cases} \quad (39)$$

Where $x_i \in [0,2\pi],\ i = 1,2$. It has 13 roots: (0.000000, 0.000000), (3.141593, 0.000000), (1.570796, 1.570796), (6.283185, 0.000000), (0.000000, 3.141593), (4.712389, 1.570796), (3.141593, 3.141593), (1.570796, 4.712389), (6.283185, 3.141593), (0.000000, 6.283185), (4.712389, 4.712389), (3.141593, 6.283185) and (6.283185, 6.283185)

(22) E22:

$$\begin{cases} x_1^2 + x_2^2 - 1.0 = 0 & (1) \\ x_3^2 + x_4^2 - 1.0 = 0 & (2) \\ x_5^2 + x_6^2 - 1.0 = 0 & (3) \\ x_7^2 + x_8^2 - 1.0 = 0 & (4) \\ 4.731\times10^{-3}x_1x_3 - 0.3578x_2x_3 - 0.1238x_1 + x_7 & \\ -1.637\times10^{-3}x_2 - 0.9338x_4 - 0.3571 = 0 & (5) \\ 0.2238x_1x_3 + 0.7623x_2x_3 + 0.2638x_1 - x_7 & \\ -0.07745x_2 - 0.6734x_4 - 0.6022 = 0 & (6) \\ x_6x_8 + 0.3578x_1 + 4.731\times10^{-3}x_2 = 0 & (7) \\ -0.7623x_1 + 0.2238x_2 + 0.3461 = 0 & (8) \end{cases} \quad (40)$$

Where $x_i \in [-1,1],\ i = 1,2,\cdots,8$. It has 16 roots as shown in Table S-1:

Table S-1 The roots of E22

x_1	x_2	x_3	x_4	x_5	x_6	x_7	x_8
0.1644	-0.9864	-0.9471	-0.3210	-0.9982	-0.0594	0.4110	0.9116
0.1644	-0.9864	-0.9471	-0.3210	-0.9982	0.0594	0.4110	-0.9116
0.1644	-0.9864	-0.9471	-0.3210	0.9982	-0.0594	0.4110	0.9116
0.1644	-0.9864	-0.9471	-0.3210	0.9982	0.0594	0.4110	-0.9116
0.1644	-0.9864	0.7185	-0.9656	-0.9980	-0.0638	-0.5278	0.8494
0.1644	-0.9864	0.7185	-0.9656	-0.9980	0.0638	-0.5278	-0.8494
0.1644	-0.9864	0.7185	-0.9656	0.9980	-0.0638	-0.5278	0.8494
0.1644	-0.9864	0.7185	-0.9656	0.9980	0.0638	-0.5278	-0.8494
0.6716	0.7410	-0.6516	-0.7586	-0.9625	-0.2711	-0.4376	0.8992
0.6716	0.7410	-0.6516	-0.7586	-0.9625	0.2711	-0.4376	-0.8992
0.6716	0.7410	-0.6516	-0.7586	0.9625	-0.2711	-0.4376	0.8992
0.6716	0.7410	-0.6516	-0.7586	0.9625	0.2711	-0.4376	-0.8992
0.6716	0.7410	0.9519	-0.3064	-0.9638	-0.2666	0.4046	0.9145
0.6716	0.7410	0.9519	-0.3064	-0.9638	0.2666	0.4046	-0.9145
0.6716	0.7410	0.9519	-0.3064	0.9638	0.2666	0.4046	-0.9145
0.6716	0.7410	0.9519	-0.3064	0.9638	-0.2666	0.4046	0.9145

The equation (5), (6), (7) and (8) are the eliminated equations. x_1 , x_4 , x_6 and x_7 are the reduced variables. We can get the following variable reduction scheme:

$$\begin{aligned} & x_1 = 0.29358520x_2 + 0.45402073 \\ & x_4 = 0.06455304x_3 - 0.02363432x_2 + 0.29342213x_2x_3 - 0.55732771 \\ & x_6 = -\frac{0.01311819(8.36820813x_2 + 12.383458)}{x_8} \\ & x_7 = 0.01591312x_2 + 0.05813982x_3 + 0.63041391x_2x_3 - 0.10712485 \end{aligned} \tag{41}$$

(23) E23:

$$\begin{cases} 4x_1^3 - 3x_1 - \cos(x_2) = 0 & (1) \\ \sin(x_1^2) - |x_2| = 0 & (2) \end{cases} \tag{42}$$

Where $x_i \in [-2,2]$, $i = 1,2$. It has 6 roots: (-0.597167, -0.349098), (-0.597167, 0.349098), (-0.442798, -0.194781), (-0.442798, 0. 194781), (0.964499, -0.801774) and (0.964499, 0.801774).

The equation (1) is the eliminated equation. x_2 is the reduced variable. We can get the following variable reduction scheme:

$$x_2 = \pm\sin(x_1^2) \tag{43}$$

(24) E24:

$$\begin{cases} x_i + \sum_{j=1}^{D} x_j - (D+1) = 0, \quad i = 1,\cdots,D-1 & (1) \\ \prod_{j=1}^{D} x_j - 1 = 0 & (2) \end{cases} \tag{44}$$

Where $x_i \in [-2,2]$, $i = 1,2,\cdots,20$ and $D = 20$. It has two roots: $(-1,\cdots,1)$ and $(0.994922,\cdots,0.994922,1.101551)$.

The equations in the equation (1) are the eliminated equations. x_k, $k = 1,2,\cdots,D-1$ are the reduced variable. We can get the following variable reduction scheme:

$$x_k = \frac{21 - x_{20}}{20}, \quad k = 1,2,\cdots,D-1 \tag{45}$$

(25) E25:

$$x_i - \cos(2x_i - \sum_{j=1}^{D} x_j) = 0, \quad i = 1,\cdots,D \tag{46}$$

Where $x_i \in [-1,1]$, $i = 1,2,\cdots,D$ and $D = 3$. It has 6 roots: (0.810561,0.810561, 0.625687), (0.810561, -0.625687, 0.810561), (-0.625687, 0.810561, 0.810561), (0.543850, 0.995778, 0.543850), (0.543850, 0.543850, 0.995778), (0.995778, 0.543850, 0.543850) and (0.739086, 0.739086, 0.739086).

The equation when $i = 1$ in Eq. (46) is the eliminated equation. x_3 is the reduced variable. We can get the following variable reduction scheme:

$$x_3 = x_1 - x_2 \pm \operatorname{acos}(x_1) \tag{47}$$

(26) E26:

$$\begin{cases} x_1^2 + x_2^2 - 2 = 0 & (1) \\ x_1^2 + \dfrac{x_2^2}{4} - 1 = 0 & (2) \end{cases} \tag{48}$$

Where $x_i \in [-2,2]$, $i = 1,2$, $D = 3$. It has four roots: (-0.816497, -1.154701), (0.816497, -1.154701), (-0.816497, 1.154701) and (0.816497, 1.154701).

The equation (1) is the eliminated equation. x_2 is the reduced variable. We can get the following variable reduction scheme:

$$x_2 = \pm\sqrt{2 - x_1^2} \tag{49}$$

(27) E27:

$$\begin{cases} e^{x_1^2 + x_2^2} - 3 = 0 & (1) \\ |x_2| + x_1 - \sin(3(|x_2| + x_1)) = 0 & (2) \end{cases} \tag{50}$$

Where $x_i \in [-2,2]$, $i = 1,2$. It has 6 roots: (-0.741152, -0.741152), (-0.741152, 0.741152), (-0.256625, 1.016246), (-0.256625, -1.016246), (-1.016246, -0.256625) and (-1.016246, 0.256625).

The equation (1) is the eliminated equation. x_2 is the reduced variable. We can get the following variable reduction scheme:

$$x_2 = \pm\sqrt{\ln 3 - x_1^2} \tag{51}$$

(28) E28:

$$\begin{cases} -3.84x_1^2 + 3.84x_1 - x_2 = 0 & (1) \\ -3.84x_2^2 + 3.84x_2 - x_3 = 0 & (2) \\ -3.84x_3^2 + 3.84x_3 - x_1 = 0 & (3) \end{cases} \tag{52}$$

Where $x_i \in [0,1]$, $i = 1,2,3$. It has 8 roots: (0.000000, 0.000000, 0.000000), (0.488122, 0.959435, 0.149452), (0.540304, 0.953754, 0.169399), (0.959447, 0.149373, 0.487917), (0.149440, 0.488092, 0.959440), (0.953781, 0.169343, 0.540157), (0.169254, 0.539937, 0.953788) and (0.739584, 0.739584, 0.739574)

The equation (1) and (2) is the eliminated equations. x_2 and x_3 are the reduced variables. We can get the following variable reduction scheme:

$$x_2 = \frac{96x_1 - 96x_1^2}{25}$$
$$x_3 = -\frac{884736x_1^4 + 1769472x_1^3 - 1115136x_1^2}{15625} + \frac{9216x_1}{625} \tag{53}$$

(29) E29:

$$\begin{cases} 3x_1^2 + \sin(x_1x_2) - x_3^2 + 2.0 = 0 & (1) \\ 2x_1^3 + x_2^2 - x_3 + 3.0 = 0 & (2) \\ \sin(2x_1) + \cos(x_2x_3) + x_2 - 1.0 = 0 & (3) \end{cases} \tag{54}$$

Where $x_1 \in [-5,5]$, $x_2 \in [-1,3]$, $x_3 \in [-5,5]$. It has two roots: (-0.064417, 2.090440, -1.370473) and (-0.032759, 1.264629, 1.400644).

The equation (1) is regarded as the eliminated equation, and x_3 is the reduced variable. We can get the following variable reduction scheme:

$$x_3 = \pm\sqrt{3x_1^2 + \sin(x_1x_2) + 2.0} \tag{55}$$

(30) E30:

$$\begin{cases} 5x_1^9 - 6x_1^5x_2^2 + x_1x_2^4 + 2x_1x_3 = 0 & (1) \\ -2x_1^6x_2 + 2x_1^2x_2^3 + 2x_2x_3 = 0 & (2) \\ x_1^2 + x_2^2 - 0.265625 = 0 & (3) \end{cases} \tag{56}$$

Where $x_1 \in [-0.6, 0.6]$, $x_2 \in [-0.6, 0.6]$, $x_3 \in [-2,5]$. It has 12 roots: (0.279855, 0.432789, -0.014189), (0.279855, -0.432789, -0.014189), (-0.279855, 0.432789, -0.014189), (-0.279855, -0.432789, -0.014189), (0.466980, 0.218070, 0.000000), (-0.466980, 0.218070, 0.000000), (0.466980, -0.218070, 0.000000), (-0.466980, -0.218070, 0.000000), (0.000000, 0.515388, 0.000000), (0.000000, -0.515388, 0.000000), (0.515388, 0.000000, -0.012446) and (-0.515388, 0.000000,-0.012446).

The equation (3) and (1) are the eliminated equations. x_1 and x_3 is the reduced variable. We can get the following variable reduction scheme:

$$x_1 = \pm\sqrt{0.265625 - x_2^2}$$
$$x_3 = (3x_1^5x_2^2 - x_1x_2^4/2 - (5x_1^9)/2)/x_1 \tag{57}$$

(31) E31:

$$\begin{cases} x_1^2 - x_2 - 2 = 0 & (1) \\ x_1 + \sin(\frac{\pi}{2}x_2) = 0 & (2) \end{cases} \tag{58}$$

Where $x_1 \in [0,1]$, $x_2 \in [-10, 0]$. It has two roots: $(0,-2)$ and $(0.707660,-1.5)$.

The equation (1) is the eliminated equation. x_2 is the reduced variable. We can get the following variable reduction scheme:

$$x_2 = x_1^2 - 2 \tag{59}$$

(32) E32:

$$\begin{cases} x_1^2 + x_2^2 + x_1 + x_2 - 8 = 0 & (1) \\ x_1|x_2| + x_1 + |x_2| - 5 = 0 & (2) \end{cases} \tag{60}$$

Where $x_1 \in [0, 2.5]$, $x_2 \in [-4,6]$. It has four roots: (0.404634, -3.271577), (2.403604, -0.762837), (1, 2) and (2, 1).

The equation (2) is the eliminated equation. x_1 is the reduced variable. We can get the following variable reduction scheme:

$$x_1 = \frac{5 - |x_2|}{|x_2| + 1} \tag{61}$$

(33) E33:

$$\begin{cases} x_1^2 - |x_2| + 1 + \frac{1}{9}|x_1 - 1| = 0 & (1) \\ x_2^2 + 5x_1^2 - 7 + \frac{1}{9}|x_2| = 0 & (2) \end{cases} \tag{62}$$

Where $x_1 \in [-1,1]$, $x_2 \in [-10,10]$. It has four roots: (-0.814326, -1.864719), (0.861828, -1.758100), (-0.814326, 1.864719) and (0.861828, 1.758100).

The equation (1) is the eliminated equation. x_2 is the reduced variable. We can get the following variable reduction scheme:

$$x_2 = \pm(x_1^2 + 1 + \frac{1}{9}|x_1 - 1|) \tag{63}$$

(34) E34:

$$\begin{cases} 0.5\sin(x_1x_2) - \frac{0.25}{\pi}x_2 - 0.5x_1 = 0 & (1) \\ (1 - \frac{0.25}{\pi})(e^{2x_1} - e) + \frac{e}{\pi}x_2 - 2ex_1 = 0 & (2) \end{cases} \tag{64}$$

Where $x_1 \in [0.25, 1]$, $x_2 \in [1.5, 2\pi]$. It has two roots: $(0.299465, 2.836948)$ and $(0.499966, 3.141589)$.

The equation (2) is the eliminated equation. x_2 is the reduced variable. We can get the following variable reduction scheme:

$$x_2 = 2\pi x_1 - (0.25 - \pi)(e^{2x_1 - 1} - 1) \tag{65}$$

(35) E35:

$$\begin{cases} x_1^{x_2}+x_2^{x_1}-5x_1x_2x_3-85=0 & (1)\\ x_1^3-x_2^{x_3}-x_3^{x_2}-60=0 & (2)\\ x_1^{x_3}+x_3^{x_1}-x_2-2=0 & (3)\end{cases} \quad (66)$$

Where $x_1 \in [3,5]$, $x_2 \in [2,4]$, $x_3 \in [0.5,2]$. It has one root: $(4,3,1)$.

The equation (3) is the eliminated equation. x_2 is the reduced variable. We can get the following variable reduction scheme:

$$x_2 = x_1^{x_3}+x_3^{x_1}-2 \quad (67)$$

(36) E36:

$$\begin{cases} x_1^3-3x_1x_2^2-1=0 & (1)\\ 3x_1^2x_2-x_2^3+1=0 & (2)\end{cases} \quad (68)$$

Where $x_1 \in [-1,\ -0.1]$, $x_2 \in [-2,2]$. It has two roots: (-0.793701, -0.793701) and (-0.290515,1.084215).

The equation (1) is the eliminated equation. x_2 is the reduced variable. We can get the following variable reduction scheme:

$$x_2 = \pm\sqrt{\frac{x_1^3-1}{3x_1}} \quad (69)$$

(37) E37:

$$\begin{cases} 0.1x_1+\cos(2x_2)+0.09240=0 & (1)\\ \sin(3x_3)+\sin(\frac{10\sin x_1}{3})+\ln(2x_2)-2.52x_3+0.08805=0 & (2)\\ 2(x_1-0.75)^2+\sin(16\pi x_2-\frac{\pi}{2})-3.26815=0 & (3)\end{cases} \quad (70)$$

Where $x_1 \in [1,2.5]$, $x_2 \in [0.2,2]$, $x_3 \in [0.1,3]$. It has one roots: (1.852100,0.926050,0.617370).

The equation (1) is the eliminated equation. x_1 is the reduced variable. We can get the following variable reduction scheme:

$$x_1 = -10\cos(2x_2)-0.924 \quad (71)$$

(38) E38:

$$\begin{cases} 4x_1^3-3x_1-x_2=0 & (1)\\ x_1^2-x_2=0 & (2)\end{cases} \quad (72)$$

Where $x_1 \in [-5,1.5]$, $x_2 \in [0,5]$. It has three roots: (-0.75,0.5625), (0,0) and (1,1).

The equation (2) is the eliminated equation. x_2 is the reduced variable. We can get the following variable reduction scheme:

$$x_2 = x_1^2 \quad (73)$$

(39) E39:

$$\begin{cases} x_1^3-3x_1x_2^2+a_1(2x_1^2+x_1x_2)+b_2x_2^2+c_1x_1+a_2x_2=0 & (1)\\ 3x_1^2x_2-x_2^3-a_1(4x_1x_2-x_2^2)+b_2x_1^2+c_2=0 & (2)\end{cases} \quad (74)$$

Where $x_1 \in [0,2]$, $x_2 \in [10,30]$ and $a_1=25$, $b_1=1$, $c_1=2$, $a_2=3$, $b_2=4$, $c_2=5$. It has two roots: (1.6359718,13.8476653) and (0.6277425,22.2444123).

The equation (2) is the eliminated equation. x_1 is the reduced variable. We can get the following variable reduction scheme:

$$x_1 = \frac{50x_2 \pm \sqrt{3x_2^4-71x_2^3+2400x_2^2-15x_2-20}}{3x_2+4} \quad (75)$$

(40) E40:

$$\begin{cases} x_1^2-x_1-x_2^2-x_2+x_3^2=0 & (1)\\ \sin(x_2-e^{x_1})=0 & (2)\\ x_3-\ln(|x_2|)=0 & (3)\end{cases} \quad (76)$$

Where $x_1 \in [0,2]$, $x_2 \in [-10,10]$, $x_3 \in [-1,1]$. It has five roots: (0.825297, -0.859034, -0.151946), (1.299490, 0.525835, -0.642769), (1.533662, -1.648068, 0.499604), (1.981360, -2.172180, 0.775731) and (1.983283, 0.983378, -0.016762)

The equation (3) is regarded as the eliminated equation. x_3 is the reduced variable. We can get the following variable reduction scheme:

$$x_3 = \ln(|x_2|) \quad (77)$$

(41) E41:

$$\begin{cases} x_1^4+4x_2^4-6.0=0 & (1)\\ x_1^2x_2-0.6787=0 & (2)\end{cases} \quad (78)$$

Where $x_1 \in [-2,2]$, $x_2 \in [0,1.1]$. It has four roots: (-1.563533, 0.277628), (0.789706, 1.088295), (1.563533, 0.277628) and (-0.789706, 1.088295).

The equation (2) is the eliminated equation. x_1 is the reduced variable. We can get the following variable reduction scheme:

$$x_1 = \pm\sqrt{\frac{0.6787}{x_2}} \quad (79)$$

(42) E42:

$$\begin{cases} \frac{0.25}{\pi}x_2 + 0.5x_1 - 0.5\sin(x_1x_2) = 0 & (1) \\ \frac{e}{\pi}x_2 - 2ex_1 + (1-\frac{0.25}{\pi})(e^{2x_1} - e) = 0 & (2) \end{cases} \tag{80}$$

Where $x_1 \in [0.25,1]$, $x_2 \in [1.5, 2\pi]$. It has two roots: $(0.5, \pi)$ and $(0.2995, 2.8369)$.

The equation (2) is the eliminated equation. x_2 is the reduced variable. We can get the following variable reduction scheme:

$$x_2 = 2\pi x_1 - (\pi - 0.25)(e^{2x_1 - 1} - 1) \tag{81}$$

(43) E43-Chemical equilibrium application:

$$\begin{cases} x_1x_2 + x_1 - 3x_5 = 0 & (1) \\ 2x_1x_2 + x_1 + x_2x_3^2 + R_8x_2 - Rx_5 + \\ 2R_{10}x_2^2 + R_7x_2x_3 + R_9x_2x_4 = 0 & (2) \\ 2x_2x_3^2 + 2R_5x_3^2 - 8x_5 + R_6x_3 + R_7x_2x_3 = 0 & (3) \\ R_9x_2x_4 + 2x_4^2 - 4Rx_5 = 0 & (4) \\ x_1(x_2+1) + R_{10}x_2^2 + x_2x_3^2 + R_8x_2 + \\ R_5x_3^2 + x_4^2 - 1 + R_6x_3 + R_7x_2x_3 + R_9x_2x_4 = 0 & (5) \end{cases} \tag{82}$$

Where $x_1, x_5 \in [0,1]$, $x_2 \in [0,60]$, $x_3, x_4 \in [-1,1]$ and $R = 10.0$, $R_5 = 0.193$, $R_6 = \frac{0.002597}{\sqrt{40}}$, $R_7 = \frac{0.003448}{\sqrt{40}}$, $R_8 = \frac{0.00001799}{40}$, $R_9 = \frac{0.0002155}{\sqrt{40}}$, $R_{10} = \frac{0.00003846}{40}$. It has infinitely many optimal solutions.

The equation (1) and (4) are the eliminated equations. x_1 and x_5 are the reduced variables. We can get the following variable reduction scheme:

$$\begin{aligned} x_1 &= \frac{3(2x_4^2 + \mathrm{R}_9x_2x_4)}{4\mathrm{R}(x_2+1)} \\ x_5 &= \frac{x_4^2}{2\mathrm{R}} + \frac{\mathrm{R}_9x_2x_4}{4\mathrm{R}} \end{aligned} \tag{83}$$

(44) E44- Neurophysiology application model:

$$\begin{cases} x_1^2 + x_3^2 = 1 & (1) \\ x_2^2 + x_4^2 = 1 & (2) \\ x_5x_3^3 + x_6x_4^3 - c_1 = 0 & (3) \\ x_5x_1^3 + x_6x_2^3 - c_2 = 0 & (4) \\ x_5x_1x_3^2 + x_6x_4^2x_2 - c_3 = 0 & (5) \\ x_5x_3x_1^2 + x_6x_2^2x_4 - c_4 = 0 & (6) \end{cases} \tag{84}$$

Where $x_i \in [-10,10]$, $i = 1,2,\cdots,D$ and $D = 6$, $c_j = 0$, $j = 1,2,\cdots,4$. It has infinitely many optimal solutions.

The equation (1), (2) and (3) are the eliminated equations. x_1, x_2 and x_6 are the reduced variables. We can get the following variable reduction scheme:

$$\begin{aligned} x_1 &= \pm\sqrt{1 - x_3^2} \\ x_2 &= \pm\sqrt{1 - x_4^2} \\ x_6 &= -x_5x_3^3 / x_4^3 \end{aligned} \tag{85}$$

(45) E45- Combustion theory application model:

$$\begin{cases} x_2 + 2x_6 + x_9 + 2x_{10} - 10^{-5} = 0 & (1) \\ x_3 + x_8 - 3\times10^{-5} = 0 & (2) \\ x_1 + x_3 + 2x_5 + 2x_8 + x_9 + x_{10} - 5\times10^{-5} = 0 & (3) \\ x_4 + x_7 - 10^{-5} = 0 & (4) \\ 0.5140437\times10^{-7}x_5 - x_1^2 = 0 & (5) \\ 0.1006932\times10^{-6}x_6 - 2x_2^2 = 0 & (6) \\ 0.7816278\times10^{-15}x_7 - x_4^2 = 0 & (7) \\ 0.1496236\times10^{-6}x_8 - x_1x_3 = 0 & (8) \\ 0.6194411\times10^{-7}x_9 - x_1x_2 = 0 & (9) \\ 0.2089296\times10^{-14}x_{10} - x_1x_2^2 = 0 & (10) \end{cases} \tag{86}$$

Where $x_i \in [-10,10]$, $i = 1,2,\cdots,D$ and $D = 10$. It has infinitely many optimal solutions.

The equation (1), (2), (3), and (4) are the eliminated equations. x_1, x_2, x_3 and x_4 are the reduced variables. We can get the following variable reduction scheme:

$$\begin{aligned} x_1 &= \frac{1}{50000} - x_8 - x_9 - x_{10} - 2x_5 \\ x_2 &= \frac{1}{100000} - x_9 - 2x_{10} - 2x_6 \\ x_3 &= \frac{3}{100000} - x_8 \\ x_4 &= \frac{1}{100000} - 2x_7 \end{aligned} \tag{87}$$

(46) E46- Economics modeling system:

$$\begin{cases} (x_k + \sum_{i=1}^{D-k-1} x_i x_{i+k}) x_D - \mathrm{c}_k = 0 \quad 1 \le k \le D-1 & (1) \\ \sum_{l=1}^{D-1} x_l + 1 = 0 & (2) \end{cases} \tag{88}$$

Where $x_i \in [-10,10]$, $i = 1,2,\cdots,D$ and $D = 5$. It has infinitely many optimal solutions.

The equation (2) is the eliminated equation. x_1 is the reduced variable. We can get the following variable reduction scheme:

$$x_1 = 1 - \sum_{j=2}^{D-1} x_j \tag{89}$$

Table S-2 The status of QR-indicator, RR-indicator and SR-indicator of DR-JADE and VR-DR-JADE, where “NaN” means not available.

Test problem	Status	QR		RR		SR	
		DR-JADE	VR-DR-JADE	DR-JADE	VR-DR-JADE	DR-JADE	VR-DR-JADE
E1	Mean	2.35E-06	**4.12E-07**	1.0000	**1.0000**	1.0000	**1.0000**
	Std Dev	1.52E-06	**8.41E-07**				
E2	Mean	5.24E-16	**9.56E-22**	1.0000	**1.0000**	1.0000	**1.0000**
	Std Dev	2.15E-15	**5.06E-21**				
E3	Mean	3.12E-15	**1.36E-16**	0.8667	**1.0000**	0.7333	**1.0000**
	Std Dev	1.11E-14	**5.18E-16**				
E4	Mean	1.05E-07	**2.10E-31**	1.0000	**1.0000**	1.0000	**1.0000**
	Std Dev	5.74E-07	**1.15E-30**				
E6	Mean	8.72E-24	**0.00E+00**	1.0000	**1.0000**	1.0000	**1.0000**
	Std Dev	4.02E-23	**0.00E+00**				
E7	Mean	7.79E-20	**6.70E-31**	1.0000	**1.0000**	1.0000	**1.0000**
	Std Dev	4.24E-19	**3.36E-30**				
E8	Mean	2.39E-20	**1.74E-24**	1.0000	**1.0000**	1.0000	**1.0000**
	Std Dev	1.13E-19	**9.52E-24**				
E9	Mean	NaN	**NaN**	0.0000	**0.0000**	0.0000	**0.0000**
	Std Dev	NaN	**NaN**				
E10	Mean	3.76E-08	**7.68E-11**	0.9970	**1.0000**	0.9667	**1.0000**
	Std Dev	1.17E-07	**4.08E-10**				
E11	Mean	7.02E-09	**1.28E-09**	0.9378	**1.0000**	0.2667	**1.0000**
	Std Dev	3.24E-08	**6.97E-09**				
E13	Mean	1.93E-06	**8.30E-12**	1.0000	**1.0000**	1.0000	**1.0000**
	Std Dev	3.24E-08	**4.03E-11**				
E14	Mean	4.05E-09	**9.32E-16**	0.9667	**1.0000**	0.7333	**1.0000**
	Std Dev	1.61E-08	**4.07E-15**				
E15	Mean	8.28E-08	**1.03E-28**	1.0000	**1.0000**	1.0000	**1.0000**
	Std Dev	4.53E-07	**8.72E-29**				
E16	Mean	8.02E-08	**4.69E-09**	1.0000	0.8571	1.0000	0.0000
	Std Dev	2.60E-07	**1.62E-08**				
E17	Mean	3.28E-13	**8.05E-31**	1.0000	**1.0000**	1.0000	**1.0000**
	Std Dev	1.19E-12	**3.56E-46**				
E18	Mean	9.78E-32	**0.00E+00**	1.0000	**1.0000**	1.0000	**1.0000**
	Std Dev	3.16E-31	**0.00E+00**				
E19	Mean	4.61E-09	**1.70E-20**	0.8600	**1.0000**	0.0333	**1.0000**
	Std Dev	1.56E-08	**9.31E-20**				
E20	Mean	3.43E-09	**3.30E-13**	1.0000	**1.0000**	1.0000	**1.0000**
	Std Dev	1.83E-08	**1.15E-12**				
E22	Mean	7.67E-11	**6.09E-27**	0.8375	**0.8729**	0.0333	**0.0667**
	Std Dev	2.90E-10	**2.69E-26**				
E23	Mean	3.51E-13	**7.11E-29**	1.0000	**1.0000**	1.0000	**1.0000**
	Std Dev	1.92E-12	**3.83E-28**				
E24	Mean	NaN	**0.00E+00**	0.0000	**1.0000**	0.0000	**1.0000**
	Std Dev	NaN	**0.00E+00**				
E25	Mean	3.16E-16	2.04E-14	1.0000	**1.0000**	1.0000	**1.0000**
	Std Dev	1.27E-15	1.12E-13				
E26	Mean	3.46E-12	**4.74E-27**	1.0000	**1.0000**	1.0000	**1.0000**
	Std Dev	1.89E-11	**2.60E-26**				
E27	Mean	3.15E-11	**9.58E-18**	1.0000	**1.0000**	1.0000	**1.0000**
	Std Dev	1.72E-10	**5.23E-17**				
E28	Mean	2.39E-07	**1.51E-32**	0.9208	**1.0000**	0.5000	**1.0000**
	Std Dev	1.90E-07	**1.54E-33**				
E29	Mean	4.93E-15	**1.55E-18**	1.0000	**1.0000**	1.0000	**1.0000**
	Std Dev	2.69E-14	**8.45E-18**				
E30	Mean	6.35E-09	**3.58E-23**	0.9306	**0.9972**	0.3667	**0.9667**
	Std Dev	1.30E-08	**1.17E-22**				
E31	Mean	1.61E-21	**2.99E-48**	1.0000	**1.0000**	1.0000	**1.0000**
	Std Dev	8.82E-21	**1.64E-47**				
E32	Mean	1.54E-14	**4.45E-20**	1.0000	**1.0000**	1.0000	**1.0000**

	Std Dev	8.41E-14	**2.43E-19**				
E33	Mean	1.11E-15	**7.39E-19**	1.0000	**1.0000**	1.0000	**1.0000**
	Std Dev	4.66E-15	**4.05E-18**				
E34	Mean	4.98E-07	**8.73E-26**	0.5000	**1.0000**	0.0000	**1.0000**
	Std Dev	7.16E-07	**4.78E-25**				
E35	Mean	2.16E-17	**3.02E-18**	1.0000	**1.0000**	1.0000	**1.0000**
	Std Dev	6.56E-17	**1.13E-17**				
E36	Mean	1.89E-19	**4.72E-29**	1.0000	**1.0000**	1.0000	**1.0000**
	Std Dev	1.03E-18	**2.58E-28**				
E37	Mean	9.56E-09	**2.31E-09**	1.0000	**1.0000**	1.0000	**1.0000**
	Std Dev	4.91E-08	**1.26E-08**				
E38	Mean	3.05E-19	**9.57E-24**	1.0000	**1.0000**	1.0000	**1.0000**
	Std Dev	1.08E-18	**4.81E-23**				
E39	Mean	8.40E-09	**2.33E-25**	1.0000	**1.0000**	1.0000	**1.0000**
	Std Dev	4.60E-08	**0.00E+00**				
E40	Mean	8.82E-09	**3.05E-09**	0.9467	**0.9867**	0.7333	**0.9333**
	Std Dev	4.81E-08	**1.67E-08**				
E41	Mean	3.73E-20	**3.94E-31**	1.0000	**1.0000**	1.0000	**1.0000**
	Std Dev	1.81E-19	**0.00E+00**				
E42	Mean	1.23E-21	**1.83E-28**	1.0000	**1.0000**	1.0000	**1.0000**
	Std Dev	6.73E-21	**7.00E-28**				

Table S-3 Root rate of the 13 compared methods on 39 test problems.

Test problem	VR-DR-JADE	DR-JADE	VR-DR-CLPSO	DR-CLPSO	VR-MONES	MONES	A-WeB	NCDE	NSDE	I-HS	GA-SQP	PSO-NM	NCSA
E1	**1.0000**	**1.0000**	**1.0000**	0.5000	**1.0000**	**1.0000**	0.7250	**1.0000**	**1.0000**	**1.0000**	0.7667	**1.0000**	0.9833
E2	**1.0000**	**1.0000**	**1.0000**	**1.0000**	**1.0000**	0.9778	0.8800	**1.0000**	0.9778	0.9667	0.7000	0.9556	0.8444
E3	**1.0000**	0.8667	0.0000	0.0000	0.0000	0.0000	0.5450	0.9897	0.9256	0.7000	0.2000	0.4000	0.3000
E4	**1.0000**	**1.0000**	**1.0000**	**1.0000**	**1.0000**	**1.0000**	**1.0000**	**1.0000**	**1.0000**	**1.0000**	0.6667	**1.0000**	**1.0000**
E6	**1.0000**	**1.0000**	**1.0000**	**1.0000**	**1.0000**	**1.0000**	0.8300	**1.0000**	**1.0000**	0.9000	0.9000	0.8333	0.0500
E7	**1.0000**	**1.0000**	**1.0000**	**1.0000**	**1.0000**	**1.0000**	**1.0000**	**1.0000**	**1.0000**	**1.0000**	0.7667	0.9111	0.9667
E8	**1.0000**	**1.0000**	**1.0000**	**1.0000**	**1.0000**	**1.0000**	**1.0000**	**1.0000**	**1.0000**	**1.0000**	0.8000	**1.0000**	0.9667
E9	0.0000	0.0000	0.0000	0.0000	**1.0000**	**1.0000**	0.6200	0.9833	**1.0000**	0.0000	0.0000	0.0000	0.5667
E10	**1.0000**	0.9970	**1.0000**	0.6485	**1.0000**	0.9879	**1.0000**	0.9848	0.9485	0.6970	0.2424	0.8182	0.6061
E11	**1.0000**	0.9378	**1.0000**	0.9867	**1.0000**	0.4889	0.9573	0.9778	0.9644	0.4844	0.2000	0.6844	0.4444
E13	**1.0000**	**1.0000**	**1.0000**	0.3000	**1.0000**	**1.0000**	**1.0000**	**1.0000**	0.9333	0.2667	**1.0000**	0.8667	**1.0000**
E14	**1.0000**	0.9667	**1.0000**	0.8917	0.6542	0.1250	0.9400	0.8708	0.8833	0.8208	0.1875	0.8750	0.5375
E15	**1.0000**	**1.0000**	**1.0000**	0.4000	**1.0000**	0.0333	0.4200	0.0667	0.1333	0.0333	0.6000	**1.0000**	**1.0000**
E16	0.8571	**1.0000**	0.8571	**1.0000**	0.8667	0.4476	0.8371	0.919	0.9238	0.6762	0.4381	0.8952	0.5333
E17	**1.0000**	**1.0000**	**1.0000**	0.5000	0.5667	0.0778	0.8933	0.0000	0.0111	0.8778	0.6333	0.6444	0.5444
E18	**1.0000**	**1.0000**	**1.0000**	0.8000	**1.0000**	**1.0000**	**1.0000**	**1.0000**	**1.0000**	**1.0000**	0.9333	0.7333	**1.0000**
E19	**1.0000**	0.8600	**1.0000**	0.8833	**1.0000**	0.4300	0.8880	0.9933	0.9533	0.7933	0.1133	0.5933	0.5200
E20	**1.0000**	**1.0000**	0.5185	0.9556	0.7778	0.3111	0.9733	0.9185	0.9333	0.7667	0.2259	0.3926	0.5222
E22	0.8729	0.8375	0.9854	0.6513	0.2146	0.0813	0.6688	0.9917	0.9896	0.8063	0.1375	0.0667	0.2125
E23	**1.0000**	**1.0000**	**1.0000**	0.9778	**1.0000**	0.6556	0.9433	0.9944	**1.0000**	0.8944	0.4889	0.8333	0.7722
E24	**1.0000**	0.0000	**1.0000**	0.0000	**1.0000**	0.0000	0.6200	0.0000	0.0000	0.0000	0.0000	0.0000	0.0333
E25	**1.0000**	**1.0000**	**1.0000**	0.9857	0.7095	0.5952	0.9514	0.9762	0.9952	0.6857	0.5286	0.8952	0.6762
E26	**1.0000**	**1.0000**	**1.0000**	**1.0000**	**1.0000**	0.5250	0.9950	**1.0000**	**1.0000**	**1.0000**	0.5000	0.6833	0.9000
E27	**1.0000**	**1.0000**	**1.0000**	**1.0000**	**1.0000**	0.5000	**1.0000**	**1.0000**	0.9944	0.9833	0.4389	0.7778	0.7222
E28	**1.0000**	0.9208	**1.0000**	0.9667	0.9875	0.7000	0.8550	**1.0000**	**1.0000**	0.4750	0.4000	0.7500	0.5500
E29	**1.0000**	**1.0000**	**1.0000**	0.9500	0.9667	0.5000	**1.0000**	**1.0000**	0.9833	0.9833	0.6167	0.8000	0.9000
E30	0.9972	0.9306	0.9944	0.0583	0.9667	0.6333	0.0933	0.9750	0.9389	0.9222	0.2833	0.4778	0.4278
E31	**1.0000**	**1.0000**	**1.0000**	0.9667	**1.0000**	**1.0000**	**1.0000**	0.7833	0.9167	0.9833	0.6333	**1.0000**	0.9167
E32	**1.0000**	**1.0000**	**1.0000**	**1.0000**	**1.0000**	0.9250	**1.0000**	0.9167	0.9917	**1.0000**	0.4667	0.8833	0.8833
E33	**1.0000**	**1.0000**	**1.0000**	**1.0000**	**1.0000**	0.5000	**1.0000**	0.4500	0.9250	**1.0000**	0.4583	0.9000	0.8583
E34	**1.0000**	0.5000	**1.0000**	**1.0000**	**1.0000**	0.9500	**1.0000**	0.0000	0.0000	0.9167	0.5500	**1.0000**	0.4333
E35	**1.0000**	**1.0000**	0.6333	0.0000	0.0667	**1.0000**	**1.0000**	0.0000	0.0000	**1.0000**	0.1333	**1.0000**	**1.0000**
E36	**1.0000**	**1.0000**	**1.0000**	**1.0000**	**1.0000**	**1.0000**	**1.0000**	**1.0000**	**1.0000**	**1.0000**	0.65	**1.0000**	0.9833
E37	**1.0000**	**1.0000**	**1.0000**	0.7667	**1.0000**	0.5000	0.8800	0.3667	0.8667	**1.0000**	0.1667	0.8000	**1.0000**
E38	**1.0000**	**1.0000**	**1.0000**	0.9778	**1.0000**	**1.0000**	**1.0000**	0.7889	0.9889	0.9556	0.7222	0.9778	0.8556
E39	**1.0000**	**1.0000**	**1.0000**	0.0000	**1.0000**	0.5000	0.9400	0.3000	0.5000	0.9333	0.0000	**1.0000**	0.7500
E40	0.9867	0.9467	0.9200	0.0267	0.8467	0.5333	0.9320	0.8067	0.8600	0.9933	0.2200	0.8400	0.7200
E41	**1.0000**	**1.0000**	**1.0000**	0.9583	0.5000	**1.0000**	**1.0000**	0.9917	**1.0000**	0.9833	0.5250	0.9000	0.8167
E42	**1.0000**	**1.0000**	**1.0000**	**1.0000**	**1.0000**	**1.0000**	0.9900	0.9833	**1.0000**	**1.0000**	0.5667	**1.0000**	0.9833
Avg.	**0.9670**	0.9170	**0.9207**	0.7218	**0.8738**	0.6661	0.8815	0.7956	0.8343	0.8077	0.4579	0.7741	0.7123

Table S-4 Success rate of of the 13 compared methods on 39 test problems.

Test problem	VR-DR-JADE	DR-JADE	VR-DR-CLPSO	DR-CLPSO	VR-MONES	MONES	A-WeB	NCDE	NSDE	I-HS	GA-SQP	PSO-NM	NCSA
E1	**1.0000**	**1.0000**	**1.0000**	0.2667	**1.0000**	**1.0000**	0.5300	**1.0000**	**1.0000**	**1.0000**	0.5333	**1.0000**	0.9667
E2	**1.0000**	**1.0000**	**1.0000**	**1.0000**	**1.0000**	0.9333	0.6800	1.0000	0.9333	0.9000	0.3667	0.8667	0.5667
E3	**1.0000**	0.7333	0.0000	0.0000	0.0000	0.0000	0.2000	0.8667	0.3667	0.5300	0.0000	0.0667	0.0000
E4	**1.0000**	**1.0000**	**1.0000**	**1.0000**	**1.0000**	**1.0000**	**1.0000**	**1.0000**	**1.0000**	**1.0000**	0.3333	**1.0000**	**1.0000**
E6	**1.0000**	**1.0000**	**1.0000**	**1.0000**	**1.0000**	**1.0000**	0.6500	**1.0000**	**1.0000**	0.8300	0.8000	0.6667	0.0000
E7	**1.0000**	**1.0000**	**1.0000**	**1.0000**	**1.0000**	**1.0000**	**1.0000**	**1.0000**	**1.0000**	**1.0000**	0.4000	0.7333	0.9000
E8	**1.0000**	**1.0000**	**1.0000**	**1.0000**	**1.0000**	**1.0000**	**1.0000**	**1.0000**	**1.0000**	**1.0000**	0.6000	**1.0000**	0.9333
E9	0.0000	0.0000	0.0000	0.0000	**1.0000**	**1.0000**	0.3600	0.9667	1.0000	0.0000	0.0000	0.0000	0.1333
E10	**1.0000**	0.9667	**1.0000**	0.0333	**1.0000**	0.9000	**1.0000**	0.8333	0.5667	0.0000	0.0000	0.0000	0.0000
E11	**1.0000**	0.2667	**1.0000**	0.8333	**1.0000**	0.0000	0.5800	0.8000	0.6333	0.0000	0.0000	0.0000	0.0000
E13	**1.0000**	**1.0000**	**1.0000**	0.3000	**1.0000**	**1.0000**	**1.0000**	**1.0000**	0.9333	0.2667	**1.0000**	0.8667	**1.0000**
E14	**1.0000**	0.7333	**1.0000**	0.3000	0.0000	0.0000	0.6000	0.3000	0.2667	0.1667	0.0000	0.2667	0.0000

E15	**1.0000**	**1.0000**	**1.0000**	0.4000	**1.0000**	0.0333	0.4200	0.0667	0.1333	0.0333	0.6000	**1.0000**	**1.0000**
E16	0.0000	**1.0000**	0.0000	**1.0000**	0.0667	0.0000	0.1200	0.4333	0.5000	0.0000	0.0000	0.3333	0.0000
E17	**1.0000**	**1.0000**	**1.0000**	0.0667	0.0667	0.0000	0.6800	0.0000	0.0000	0.6667	0.0667	0.0667	0.1000
E18	**1.0000**	**1.0000**	**1.0000**	0.8000	**1.0000**	**1.0000**	**1.0000**	**1.0000**	**1.0000**	**1.0000**	0.9333	0.7333	**1.0000**
E19	**1.0000**	0.0333	**1.0000**	0.1000	**1.0000**	0.0000	0.2800	0.9333	0.8000	0.0000	0.0000	0.0000	0.0000
E20	**1.0000**	**1.0000**	0.0000	0.6667	0.0000	0.0000	0.7600	0.4000	0.4667	0.0000	0.0000	0.0000	0.0000
E22	0.0667	0.0333	0.7667	0.0000	0.0000	0.0000	0.0000	0.8667	0.9000	0.0000	0.0000	0.0000	0.0000
E23	**1.0000**	**1.0000**	**1.0000**	0.8667	**1.0000**	0.0000	0.6600	0.9667	**1.0000**	0.4667	0.0000	0.2000	0.0667
E24	**1.0000**	0.0000	**1.0000**	0.0000	**1.0000**	0.0000	0.2400	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000
E25	**1.0000**	**1.0000**	**1.0000**	0.9333	0.0000	0.0000	0.7000	0.8333	0.9667	0.0333	0.0000	0.3333	0.0000
E26	**1.0000**	**1.0000**	**1.0000**	**1.0000**	**1.0000**	0.0000	0.9800	**1.0000**	**1.0000**	**1.0000**	0.0333	0.2000	0.6333
E27	**1.0000**	**1.0000**	**1.0000**	**1.0000**	**1.0000**	0.0000	**1.0000**	**1.0000**	0.9667	0.9000	0.0000	0.2667	0.0667
E28	**1.0000**	0.5000	**1.0000**	0.7333	0.9000	0.0000	0.1400	**1.0000**	**1.0000**	0.0000	0.0000	0.1333	0.0000
E29	**1.0000**	**1.0000**	**1.0000**	0.9000	0.9333	0.0000	**1.0000**	**1.0000**	0.9667	0.9667	0.2333	0.6000	0.8000
E30	0.9667	0.3667	0.9333	0.0000	0.6000	0.0000	0.0000	0.7667	0.4667	0.2667	0.0000	0.0000	0.0000
E31	**1.0000**	**1.0000**	**1.0000**	0.9333	**1.0000**	**1.0000**	**1.0000**	0.5667	0.8333	0.9667	0.3667	**1.0000**	0.8333
E32	**1.0000**	**1.0000**	**1.0000**	**1.0000**	**1.0000**	0.7000	**1.0000**	0.6667	0.9667	**1.0000**	0.0000	0.5333	0.5667
E33	**1.0000**	**1.0000**	**1.0000**	**1.0000**	**1.0000**	0.0000	**1.0000**	0.0333	0.7667	**1.0000**	0.0000	0.6000	0.5000
E34	**1.0000**	0.0000	**1.0000**	**1.0000**	**1.0000**	0.9000	**1.0000**	0.0000	0.0000	0.8333	0.3667	**1.0000**	0.0000
E35	**1.0000**	**1.0000**	0.6333	0.0000	0.0667	**1.0000**	**1.0000**	0.0000	0.0000	**1.0000**	0.1333	**1.0000**	**1.0000**
E36	**1.0000**	**1.0000**	**1.0000**	**1.0000**	**1.0000**	**1.0000**	**1.0000**	**1.0000**	**1.0000**	**1.0000**	0.4333	**1.0000**	0.9667
E37	**1.0000**	**1.0000**	**1.0000**	0.7667	**1.0000**	0.5000	0.8800	0.3667	0.8667	**1.0000**	0.1667	0.8000	**1.0000**
E38	**1.0000**	**1.0000**	**1.0000**	0.9333	**1.0000**	**1.0000**	**1.0000**	0.4333	0.9667	0.8667	0.2333	0.9333	0.5667
E39	**1.0000**	**1.0000**	**1.0000**	0.0000	**1.0000**	0.0000	0.8800	0.0000	0.0000	0.8667	0.0000	**1.0000**	0.5000
E40	0.9333	0.7333	0.6000	0.0000	0.5333	0.0000	0.6600	0.3333	0.5000	0.9667	0.0000	0.4000	0.1667
E41	**1.0000**	**1.0000**	**1.0000**	0.8333	0.0000	**1.0000**	**1.0000**	0.9667	**1.0000**	0.9333	0.0333	0.6667	0.3333
E42	**1.0000**	**1.0000**	**1.0000**	**1.0000**	**1.0000**	**1.0000**	0.9900	0.9666	**1.0000**	**1.0000**	0.4000	**1.0000**	0.9667
Avg.	**0.9222**	0.8043	**0.8701**	0.6068	**0.7479**	0.4607	0.7177	0.6761	0.7120	0.6015	0.2060	0.5197	0.4248